

*ARMY RESEARCH LABORATORY*



# GPS for Land Combat Applications

Summary Report of Army Workshop  
held at University of North Carolina,  
2 and 3 August 1995

Thomas B. Bahder, editor

ARL-SR-40

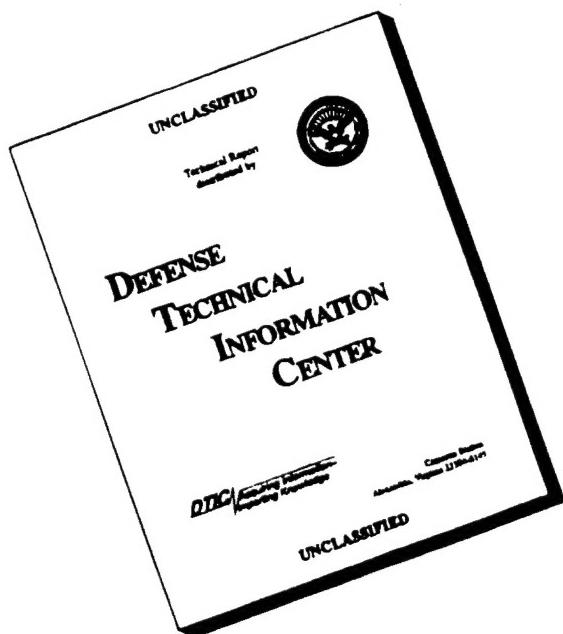
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<p>This document is a summary of the proceedings of the Army technical workshop on the Global Positioning System (GPS), which was held at the University of North Carolina on 2 and 3 August 1995. The workshop was jointly sponsored by the U.S. Army Missile Command, Army Research Laboratory, Army Research Office, and Office of the Deputy Assistant Secretary of the Army for Research and Technology. This report contains the agenda of talks, technical problems submitted for discussion by the participants, and summaries of the discussions in the 11 topical discussion groups. In addition, personal statements from several of the participants are included.</p>			
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## Preface

The Global Positioning System (GPS) is one of the great scientific and technological achievements of our time. The Army's growing interest in using the GPS is a recognition of this fact. In the near future, the Army will be the single biggest military user of the GPS.

Up to this point, the Army has had a disproportionately small involvement in issues dealing with the GPS. Bill McCorkle, Technical Director, U.S. Army Missile Command, recognized the significance of GPS for Army applications and has taken an interest in its use in future Army systems. I became involved with the GPS when questions came up regarding issues of GPS position accuracy. Because of the enormous complexity of the GPS, I decided to get a group of experts together from various disciplines important for GPS operations and applications. This idea for a GPS workshop was supported by Bill McCorkle, Bill Howard (then from the Advanced Concepts and Space (SARD-TC) Office of the Deputy Assistant Secretary of the Army for Research and Technology), and John Lyons, Director of the Army Research Laboratory.

I take this opportunity to thank Bob Guenther, Henry Everitt, and Pat Lassiter from the Army Research Office for organizing the hotel arrangements, meeting rooms at University of North Carolina, and transportation between the hotel and University. I thank Henry Everitt for help in planning the structure of the workshop. Further thanks are due to Henry Everitt and Pat Lassiter for their work during the workshop, which made everything run smoothly. I also thank Jon Dowling for his support in this work.

My thanks also go to Paul Jacobs for writing an overview that was incorporated into the introduction of this report.

I thank all the participants who gave talks at the workshop. These talks were essential to ignite the group discussions. Finally, I thank all the workshop participants for contributing their expertise, their time, and their precious travel funds to make the Army GPS workshop a success.

Tom Bahder

# Contents

<b>Preface .....</b>	<i>iii</i>
<b>1. Introduction .....</b>	1
<b>2. Workshop Overview and Agenda .....</b>	3
<b>3. Workshop Participants .....</b>	6
<b>4. Discussion Groups .....</b>	7
<b>5. Technical Issues and Discussion Group Summaries.....</b>	8
<b>Army Requirements — August 2: Wednesday Morning Session .....</b>	9
Group 1   Receiver Performance in Real Systems .....	9
Group 2   Application Integration of GPS and INS.....	10
Group 3   DGPS versus GPS .....	12
<b>GPS Performance — August 2: Wednesday Afternoon Session .....</b>	13
Group 4   Receivers .....	13
Group 5   Direct Y-Code Acquisition/Algorithms .....	14
Group 6   Jamming .....	16
<b>Time Transfer and Relativity — August 3: Thursday Morning Session .....</b>	18
Group 7   Relativistic Effects .....	18
Group 8   GPS Accuracy Issues .....	20
Group 9   Clocks (Part I) .....	22
<b>Implementation Issues — August 3: Thursday Afternoon Session .....</b>	24
Group 10   Earth Models and Orbit Determination .....	24
Group 11   Clocks (Part II) .....	25
Group 12   GPS Policy, JPO Policy, and Funding of Future Work .....	27
<b>6. Conclusion .....</b>	30
<b>Acronyms .....</b>	31
<b>Appendix A. Position Accuracy Specifications .....</b>	33
<b>Appendix B. Participants' Personal Statements.....</b>	37
<b>Distribution .....</b>	75

# 1. Introduction

Reliable and accurate navigation is an essential capability for the U.S. Army to carry out its mission. The Global Positioning System (GPS) is recognized as an important tool for all-weather navigation on land, on sea, and in the air.<sup>1,2</sup> Integrating GPS into Army applications may lead to reduced costs and provide improved navigation accuracy, if GPS can satisfy the technical requirements.<sup>3</sup> Typically, these requirements fall into two categories: navigation accuracy and reliability (such as resistance to jamming by hostile forces).

Highly accurate navigation is required by certain Army applications, such as guided missiles and precision-guided munitions. The original position accuracy specification for the precise positioning service (PPS) of GPS was 16 m SEP (spherical error probable).<sup>\*</sup> (Position accuracy is specified in several ways, depending on the application; see app A for a discussion of the terminology.) Over the past several years, the accuracy has been consistently improving, and currently the accuracy of the GPS exceeds the original design specifications. Actual position accuracy depends on the user equipment and satellite geometry at the time of the measurement. Varying position accuracy estimates exist, such as 16.4 m (2 drms)<sup>4</sup> and 7.2 m (3-d rms)<sup>5</sup> (see app A for definitions of 2 drms and 3-d rms). However, for applications such as precision-guided munitions, mine warfare, and precision aircraft landings, such accuracies are still insufficient. An order of magnitude improvement in GPS accuracy is desirable for these applications.<sup>4</sup> An enhancement to the GPS, referred to as differential GPS (DGPS), can achieve such an improved accuracy, but at a substantial increase in cost. Furthermore, a portable DGPS requires additional logistics support to remote areas of operation. Therefore, if at all feasible, it would be preferable to use an already in-place system such as GPS, with no enhancements.

Since the precision of GPS has been steadily increasing with time, the following question arises: what is the time scale for order of magnitude improvement in GPS position accuracy? The answer to this question has bearing on decisions being made now and in the future on the design and procurement of future Army systems. For military applications, the previously mentioned issue of jamming must also be explored in detail.

<sup>1</sup> GPS Navstar, *Global Positioning System: User's Overview* (prepared by ARINC Research Corp. for the Program Director, Navstar Global Positioning System Joint Program Office), YEE-82-009D, March 1991.

<sup>2</sup> For an overview of the GPS see, for example, B. Hofmann-Wellenhof, H. Lichtenegger, and J. Collins, *Global Positioning System Theory and Practice*, Springer-Verlag, New York, 1993.

<sup>3</sup> W. E. Howard, white paper study, "The Evolution of GPS Accuracy: Implications for Army Operations."

<sup>4</sup> *The Global Positioning System, A Shared National Asset, Recommendations for Technical Improvements and Enhancements*, National Academic Press, 1995. Committee on the Future of the Global Positioning System, Commission on Engineering and Technical Systems, National Research Council.

<sup>5</sup> Bryant Winn, presentation at Army GPS workshop, August 1995.

\* A list of acronyms used in this report is given at the end of the main text.

## *Introduction*

It was decided that the best way to address these complex issues is to bring together in a workshop setting three groups of people: Army users of GPS, the GPS community, and people from basic science areas that are important for GPS operation. The resulting workshop was jointly sponsored by the U.S. Army Missile Command, Army Research Laboratory, Army Research Office, and Office of the Deputy Assistant Secretary of the Army for Research and Technology. The purpose of this workshop was to examine future Army requirements for GPS use, especially since the Army will be the major military user of the system. Within the limited time available in a two-day conference, the workshop also focused specifically on the additional capabilities that could be realized for Army use by improvement to the GPS and to recommend areas of investment to realize these improvements.

The issues identified included performance requirements, receiver costs and packaging requirements, integration, and policy issues, including security implementation and commercial versus military usage. Performance issues included accuracy, system response time, jam resistance, and tracking robustness. Army-unique requirements were also identified. Major discussions were held on the implementation of corrections for general and special relativity effects. It was concluded that positional accuracies on the order of 3 m (3-d rms) (stand-alone) could be achieved in the near future with little additional investment above what is currently planned, but further increases in accuracy would be significantly more expensive. In addition, we need to develop substantial low-cost antijam capability to make the GPS robust in many Army applications.

This report is a summary of the proceedings of the workshop, which was entitled "GPS for Land Combat Applications," hereafter referred to as the Army GPS workshop. This report contains the workshop outline and summaries of technical discussions that were held at the Physics Department of the University of North Carolina, Chapel Hill, on 2 and 3 August 1995.

## 2. Workshop Overview and Agenda

The Army GPS workshop hosted approximately 40 experts from a wide range of multi-disciplinary fields. The purpose of the workshop was to bring together the Army application-oriented GPS users with the basic researchers and engineers who created GPS and study it. These fell into three groups: first, experts from the GPS community, which included people with expertise in such areas as satellite navigation systems and supporting areas such as geodesy. This group included some of the original founders of GPS. The second group consisted of experts in time transfer and clocks, since GPS is basically a time-of-flight system. This group included experts in relativity, whose effects must be taken into account for the accurate operation of GPS. The third group consisted of Army users of GPS, such as integrators of inertial navigation and GPS.

The number of workshop participants was purposely kept small so that the meeting could be in the form of a workshop rather than a conference. The workshop participants are listed in section 3. The list gives the participants' names, affiliations, telephone numbers, and brief descriptions of their association with GPS.

Discussion groups were defined based on what were perceived as the important subject areas for GPS. For each discussion group, a moderator and a secretary was assigned. The role of the moderator was to keep the discussion fruitful, and to break up any fist fights that might develop because of differing opinions. The role of the secretary was to record the content of the discussion in the group. Each discussion group had an assigned topic area. These discussion groups and their topical areas are listed in section 4.

In the month preceding the workshop, the prospective workshop participants were asked to submit lists of issues for discussion at the workshop. These issues were divided among 12 topics, each of which was to be the subject of a discussion group. The groups were instructed that these issues were intended to stimulate discussion, rather than to limit it. The issues submitted by the prospective participants and assigned to the discussion groups are given in section 5.

On each of the two workshop days, morning and afternoon sessions were held. Each session began with several short talks (see the agenda at the end of this section). During the talks, sign-up sheets were passed around for the participants to choose one of three discussion groups. For each of the three discussion groups, the sign-up sheet listed the issues previously submitted for discussion by the participants. After the talks in each session were fin-

## *Workshop Overview and Agenda*

ished, participants were asked to join their respective discussion groups, in separate rooms. Finally, the three discussion groups came back together and the moderator from each group gave a brief summary of the discussion in each group.

The secretary from each discussion group was asked to write up a one-page summary describing the group's deliberations. These summaries are given in section 5. (There is no summary for discussion group 4, because it was cancelled. During this session, people attended either group 5 or 6.)

The presentations and discussions at the workshop were unclassified.

The agenda was presented as shown opposite:

## *Workshop Overview and Agenda*

A Technical Workshop Sponsored by the Army Research Office,  
U.S. Army Missile Command,  
Army Research Laboratory,  
and  
Office of the Deputy Assistant Secretary of the Army for Research and Technology

*August 2 and 3, 1995, at University of North Carolina, Chapel Hill*

### **Workshop Agenda**

#### **August 2: Wednesday Morning Session — Army Requirements**

08:30–08:40	Welcome and Introduction of Keynote Speaker — William C. McCorkle (MICOM)
08:40–09:00	Keynote Address: Background Remarks Leading to this Conference — William E. Howard (Office of the Asst. Sec. of the Army (RDA))
09:00–10:00	GPS Aided Missile Guidance Systems — Albert Killen (MICOM) GPS Missile Guidance Issues — Brian Baeder (MICOM) Army Applications of GPS — Paul Olson (CECOM)
10:00–10:15	Break
10:15–11:45	Discussion Groups
11:45–12:00	Summary of Discussion Groups
12:00–13:30	Lunch Break

#### **August 2: Wednesday Afternoon Session — GPS Performance**

13:30–15:00	JPO Overview of GPS — Capt. Lynn Anderson (AF JPO) System Overview and Error Budgets — William Feess and Bryant Winn (Aerospace Corp.) GPS Receivers and Antennas — Daryl Thornburg (Magnavox) Orbit Determination — James O'Toole (NSWC)
15:00–15:15	Break
15:15–16:45	Discussion Groups
16:45–17:00	Summary of Discussion Groups

#### **August 3: Thursday Morning Session — Time Transfer and Relativity**

08:30–09:50	Time Transfer and Metrology — Bill Klepczynski (USNO) Correlations in GPS and USNO Clock Time — Thomas Bahder (ARL) Relativistic Effects — Neil Ashby (U of Colorado) Evidence for the Relativity of Simultaneity in Measured Pseudoranges — Carroll Alley (U of Maryland)
09:50–10:00	Break
10:00–11:45	Discussion Groups
11:45–12:00	Summary of Discussion Groups
12:00–13:30	Lunch Break

#### **August 3: Thursday Afternoon Session — Implementation Issues**

13:30–14:30	Atomic Clocks — Steve Jefferts (NIST) Earth Models — Stephen Malys (DMA)
14:30–14:45	Break
14:45–15:45	Discussion Groups
15:45–16:45	Summary of Discussion Groups
16:45–17:00	Closing Remarks — William C. McCorkle (MICOM)

Contact Person: Thomas B. Bahder, Army Research Laboratory, tel. 301-394-2042, bahder@arl.mil

### 3. Workshop Participants

The following table lists the participants in the Army GPS workshop.

Name	Affiliation	Telephone	Notes
Alley, Carroll	University of Maryland	301-405-6098	experimental relativity/GPS
Anderson, CPT Lynn	Air Force Space Command	310-363-1203	GPS Advanced Projects
Ashby, Neil	University of Colorado	303-492-4980	relativistic effects
Baeder, Brian	Missile Command (MICOM)	205-876-8046	coupled IMU (inertial measurement unit) -GPS
Bahder, Thomas B.	Army Research Laboratory (ARL)	301-394-2042	Army GPS workshop coordinator
Beard, Ronald	Naval Research Laboratory	202-767-2595	GPS/Transit
Crane, Del	Army	310-363-0595	Army GPS integration
Dowling, J.	MICOM	205-842-2800	quantum optics
Everitt, Henry	Army Research Office (ARO)	919-549-4369	laser physics
Feess, Bill	Aerospace Corp.	310-336-4397	GPS program office
Fliegel, Henry	Aerospace Corp.	310-336-1710	GPS/radiation pressure/clocks
Francisco, Sherman	Loral Inc.	301-240-5927	Former GPS Chief Engineer
Graham, Greg	MICOM	205-876-7366	coupled IMU-GPS
Greenspan, Richard	Draper Labs	617-258-4041	GPS military applications
Howard, William E.	Army	703-695-1447	Pentagon
Jacobs, Paul	MICOM	205-876-4640	Assoc. Dir. Technology
Jefferts, Steve	National Institute of Science and Technology (NIST)	303-497-7377	clocks
Johnston, Kenneth	U.S. Naval Observatory (USNO)	202-653-1513	Science Director at USNO
Killen, Albert	MICOM	205-955-8171	coupled IMU-GPS
Klepczynski, William	USNO	202-653-1521	time transfer/metrology
Malys, Steve	Defense Mapping Agency (DMA)	301-227-5186	post-fit GPS orbits
McCorkle, W. C.	MICOM	205-876-3322	Director of Missile RDEC
Misner, Charles W.	University of Maryland	301-405-6026	general relativity
Oimoen, Daniel C.	Topographic Engineering Center (TEC)	703-355-2799	GPS/Army/TEC
O'Toole, Jim	Naval Surface Warfare Center	703-663-8409	orbits/laser ranging
Olson, Paul	Communications-Electronics Command (CECOM)	908-427-3912	coupled inertial navigation system (INS) and GPS
Skatrud, David	ARO	919-549-4361	Physics Division, ARO
Thornburg, Daryl	Magnavox Inc.	310-618-7025	receivers/anti-jamming
Van Flandern, Thomas	University of Maryland	301-405-6098	celestial mechanics
Vig, John	ARL	908-427-4275	crystal clocks
Wiles, George	ARL	301-394-2621	GPS guided fuze
Winfield, David	Loral Inc.	301-240-6379	Kalman filter
Winkler, Germot	USNO	301-299-6126 (h)	time/GPS/time transfer
Winn, Bryant	Aerospace Corp.	310-336-7452	GPS program office
Wu, Andy	Aerospace Corp.	310-336-0437	GPS/clocks/INS
Yilmaz, Hüseyin	Hamamatsu Corp./Tufts University	617-729-7856	relativity

## 4. Discussion Groups

Twelve discussion groups were formed. Group 4 was cancelled.

### Army Requirements — August 2, Wednesday Morning Session

<i>Group 1</i>	<i>Receiver Performance in Real Systems</i>
	Moderator: William McCorkle      Secretary: Jon Dowling
<i>Group 2</i>	<i>Application Integration of GPS and INS (Inertial Navigation System)</i>
	Moderator: Paul Jacobs      Secretary: George Wiles
<i>Group 3</i>	<i>DGPS versus GPS</i>
	Moderator: Bryant Winn      Secretary: Steve Malys

### GPS Performance — August 2, Wednesday Afternoon Session

<i>Group 4</i>	<i>Receivers</i>
	Moderator: Paul Olson      Secretary: Daniel Oimoen
<i>Group 5</i>	<i>Direct Y-Code Acquisition/Algorithms</i>
	Moderator: Albert Killen      Secretary: Richard Greenspan
<i>Group 6</i>	<i>Jamming</i>
	Moderator: Kenneth Johnston      Secretary: Kenneth Johnston

### Time Transfer and Relativity — August 3, Thursday Morning Session

<i>Group 7</i>	<i>Relativistic Effects</i>
	Moderator: Bill McCorkle      Secretary: Tom Bahder
<i>Group 8</i>	<i>GPS Accuracy Issues</i>
	Moderator: Brian Baeder      Secretary: Albert Killen
<i>Group 9</i>	<i>Clocks (Part I)</i>
	Moderator: John Vig      Secretary: Steve Jefferts

### Implementation Issues — August 3, Thursday Afternoon Session

<i>Group 10</i>	<i>Earth Models and Orbit Determination</i>
	Moderator: Steve Malys      Secretary: Jim O'Toole
<i>Group 11</i>	<i>Clocks (Part II)</i>
	Moderator: Gernot Winkler      Secretary: John Vig
<i>Group 12</i>	<i>GPS Policy, Joint Program Office (JPO) Policy, and Funding of Future Work</i>
	Moderator: Bill Howard      Secretary: Bill Klepczynski

## **5. Technical Issues and Discussion Group Summaries**

As described in section 2, approximately one month before the workshop a request was sent to the prospective participants to submit a list of issues for discussion at the workshop. The issues were categorized into 12 areas for discussion. In the following, the submitted issues are shown according to discussion group; when appropriate, credit is given to the person(s) who submitted the given issue.

The summary from each discussion group, as written by the secretary (and in some cases the moderator), is given for each topic. These summaries are informal and are minimally edited. If clarification is sought, please note the moderator and secretary of the group, and consult section 3 for contact telephone numbers.

Although 12 discussion groups were originally planned for the workshop, group 4 was cancelled, so 11 discussion groups were actually held. (On the afternoon of August 2, only groups 5 and 6 were held, and participants attended one of these groups.)

## August 2: Wednesday Morning Session — Army Requirements

### **Group 1    *Receiver Performance in Real Systems***

Moderator: Bill McCorkle

Secretary: Jon Dowling

#### **Suggested Topics**

1. In actual usage, as established in field tests and usage of Army systems, what limits the accuracy and utility of GPS?
2. What possible improvements could be made to the various segments of the GPS, such as receivers, antennas, satellite signal strength, etc, to overcome the problems in question 1?
3. (Tom Bahder) The National Research Council (NRC) has recently completed an extensive study of possible improvements to the GPS and its augmentations for civilian and military use.<sup>4</sup> If the recommendations of the NRC are accepted, the study claims that many applications may be satisfied by the use of stand-alone GPS, without the need for DGPS. The study claims that if the recommendations of the NRC are accepted (see table 3-11 and p 102 of the NRC study), PPS horizontal accuracy will be 4.2 m (2 drms) or 1.8 m circular error probable (CEP). The study further claims that this accuracy is sufficient for the application area of precision-guided munitions. Is this accuracy sufficient for many Army applications?
4. (Brian Baeder) Environment: spinning body effects, clock stability at high dynamics, temperature, vibration, shock.
5. (Richard Greenspan) How will the field Army *train* its forces to use GPS under hostile conditions, including jamming? In particular, how will we execute the discipline to be simultaneously jamming the L1 frequency, while operating the L2 frequency? How will we avoid jamming our own receivers on L2?
6. (Richard Greenspan) How will we locate targets in GPS coordinates for hand-off to smart munitions that use GPS/INS guidance?

#### **Summary**

Gernot Winkler stressed the point repeatedly that receivers that used both GPS and GLONASS (the Russian global navigation satellite system) could be made at little extra cost, giving extra redundancy. GLONASS, however, is more susceptible to outside interference, because it relies on multiple fre-

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<sup>4</sup> *The Global Positioning System, A Shared National Asset, Recommendations for Technical Improvements and Enhancements*, National Academic Press, 1995. Committee on the Future of the Global Positioning System, Commission on Engineering and Technical Systems, National Research Council.

## *Application Integration of GPS and INS*

quencies. Both Winkler and Vig discussed the question of receiver and clock-crystal accuracy. Vig claims that there is a method to construct crystals that are immune to missile launch-phase vibrations — an important point for MICOM applications. The difference between code-taking receivers and carrier receivers was discussed: carrier tracking can improve code acquisition when the receiver is going through periods of degraded signal. Alley brought up the point of using Rb or Cs atomic clocks in receivers, but Beard thought these would be more susceptible to the g-forces and vibrations of missile launch than crystal clocks. Thornburg initiated a discussion of receiver modeling of ionospheric, tropospheric, and multipath errors. He thought the atmospheric errors could be reduced substantially — leaving multipath as the major source of error. Receivers that measure local humidity can correct for the “wet component” of the troposphere. Possible cures to the multipath problem are narrowing the time-of-arrival window and using phase information.

### **Group 2    *Application Integration of GPS and INS***

Moderator: Paul Jacobs

Secretary: George Wiles

#### **Suggested Topics**

1. From an applications point of view, what are the biggest problems in integration of GPS and INS in real Army systems?
2. What improvements in the different segments of GPS, signal in space (SIS), operational control segment (OCS), and user equipment, could be made to mitigate the above problems?
3. What (basic or applied) research work would be helpful to mitigate the problems in question 1?
4. (Andy Wu) Applications of the lightweight, small-size, and low-power atomic frequency standard for GPS receivers. Benefits of integrating the atomic frequency standard with the GPS receiver and INS.

#### **Summary**

The first morning session followed introductory comments by Bill McCorkle about current trends of force projection, and GPS's impact on digitizing the battlefield, terminal homing, and targeting. The keynote address by Bill Howard revealed that in terms of revenue for the OEMs (original equipment manufacturers), military GPS is a scant 2 percent of the total marketplace. While improvements in lethality can be had through a combination of delivery accuracy and target location error, it is up to DoD to provide for the enabling technologies that are not already being pursued in the commercial

world. A briefing on GPS guidance for ATACMS (Army Tactical Missile System) and Guided MLRS (Multiple Launch Rocket System) followed.

With this fresh in our minds, we broke off to address the problems of integration of GPS and INS in real Army systems, and the basic and applied research needed to address those problems. After a round of discussion on the more challenging applications of GPS for missiles and artillery, we listed the issues in terms of performance, cost, packaging, implementation, and policy. It was clear from our discussions that direct, fast acquisition of the Y-code (encrypted P-code—precision code) signal in the presence of jamming in a severe physical environment (spinning, high shock and vibration, extreme temperature) is the critical problem in Army standoff weapons. Also, it was recognized that early acquisition of P-code GPS allowed for a savings in cost and complexity of associated inertial measurement units (IMUs). Likewise, an enhanced jammer to signal (J/S) capability would permit GPS to supplement the IMU closer to the target area, with a similar cost savings in IMUs.

Our group identified three focus areas for research to address the technology shortfall from the commercial sector. The first is for receiver clocks to provide an accurate and stable reference, either through the high vibration of a missile boost or the extreme setback acceleration of an artillery launch. Clock technology is key to the general problem of direct Y-code acquisition. A second area is the hardware and algorithms needed for fast acquisition with a poor clock and in the presence of jamming. A goal of 1 to 10 s for a time to first fix was identified. The third area is in antijam technologies, such as improved CRPA (controlled radiation pattern antenna—null steering) technology, temporal filtering, improved satellite power, and carrier-aided tracking. In all three areas, advancements in miniaturization, power management, and packaging are needed to enable use in standoff weapons. These themes were repeated and refined throughout the course of the workshop.

### ***Group 3 DGPS versus GPS***

Moderator: Bryant Winn

Secretary: Steve Malys

#### **Suggested Topics**

1. If GPS positioning accuracy (both C/A (coarse acquisition) and P-code) could be significantly improved in the near future, what level of accuracy (specified in SEP, CEP, or 2 drms) would be acceptable for most current and near-term future Army applications? The purpose of this question is to ascertain the level of improvement in GPS position accuracy that would satisfy most Army requirements, and thereby forestall the need to implement DGPS. The

## DGPS versus GPS

NRC study<sup>4</sup> predicts that if the committee's recommendations are implemented, then the stand-alone accuracy of the SPS (standard positioning service) will be 11.1 m (2 drms), and the accuracy of the PPS will be 4.2 m (2 drms). Also, note that improvements to GPS integrity monitoring are predicted if many of the NRC study recommendations are implemented.

2. What are the advantages (other than improved accuracy) of using DGPS rather than stand-alone GPS? What applications absolutely require GPS integrity monitoring?
3. (Bryant Winn) Potential GPS improvement: User range error (URE) contribution < 1 m (rms).
4. (Richard Greenspan) Does the Army need RAIM (receiver autonomous integrity monitoring—a means by which the GPS receiver checks on the integrity of the received GPS signals in space)?

## Summary

Group made no recommendation to deploy local-area DGPS at this time.

Many recognized that the GPS is a DGPS—a wide-area DGPS. That is, users navigate with respect to orbits defined with respect to GPS/OCS monitor stations.

Most were aware of the signal in space (SIS) 1–2 m (rms) two-dimensional (2-d) navigation accuracies achieved by Malys and Feess, who independently had used an expanded GPS/OCS monitor station network and a single filter estimation process.

Most agreed that user equipment error contributions would have to be reduced for such SIS accuracies to be fully exploited.

One DoD representative indicated (1) that a proliferation of local-area differential systems was an unacceptable DoD option at this time; (2) that the Joint Chiefs of Staff (JCS) Master Navigation Plan (MNP) 1-m (95 percent) 2-d navigation accuracy was not a current driver; and (3) that 1–2 m (66 percent) 2-d navigation accuracy should be pursued without resorting to local area DGPS proliferations.

Hence, we recommend pursuit of GPS 2-d navigation accuracies at the 1–2 m (rms) level through refinements to the GPS/OCS and user equipment.

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<sup>4</sup> *The Global Positioning System, A Shared National Asset, Recommendations for Technical Improvements and Enhancements*, National Academic Press, 1995. Committee on the Future of the Global Positioning System, Commission on Engineering and Technical Systems, National Research Council.

## August 2: Wednesday Afternoon Session — GPS Performance

### Group 4    *Receivers*

Moderator: Paul Olson

Secretary: Daniel Oimoen

#### Suggested Topics

1. (Paul Olson) GPS receivers are currently being specified to have very high dynamic ranges (10–100 GHz) and very quick/sophisticated signal acquisition requirements. For this reason, manufacturers are planning to incorporate hundreds (possibly soon thousands) of correlators on what is termed “standard” GPS cards. This has an impact on the size and cost of the receivers. Many Army applications require smaller sizes without the high dynamic range. To what degree can the number of correlators be lowered, allowing for size/cost reduction, while the dynamic range capability is lowered, and signal acquisition requirements are still maintained?
2. (Dan Oimoen) One concern is the issue of GPS hardware and software to perform carrier phase processing in a jamming environment. What are the hardware options, software options, and jamming levels?
3. (Dan Oimoen) What are the merits of carrier phase processing? When is it useful? What are the hardware options, software options, and jamming levels?
4. (Brian Baeder) Antenna issues: switching network, high-temperature effects.
5. (Richard Greenspan) How should one-site GPS receivers minimize radio frequency interference (RFI), multipath distortion, and limited line-of-sight visibility to satellites? Recall the anecdotal information about an *L*-band troposcatter transmitter creating severe RFI for GPS users in its vicinity.

#### Summary

This session was cancelled.

**Group 5    Direct Y-Code Acquisition/Algorithms**

Moderator: Albert Killen

Secretary: Richard Greenspan

**Suggested Topics**

1. (George Wiles) Under direct Y-code acquisition, what are the best options for external time aiding? This refers to locking on to the nonrepeating secure signal without first acquiring the repeating C/A code, getting the system time, and handing over to the Y-code. This becomes an issue if the civilian signal is "denied" (switched off, jammed, whatever) and not available for handover.
2. (Tom Bahder) The recent NRC study<sup>4</sup> recommends that a new signal, an L4, should be added to the satellite broadcast signals. This would allow the civilian community to make ionospheric corrections based on the L1 and L4 frequencies. The NRC study then suggests that during adverse times, the military could intentionally jam the L1 and L4 signals, but that the L2 signal could still be used by military receivers capable of direct Y-code acquisition. What impact will adding an L4 frequency have?
3. What areas need research for developing inexpensive receivers capable of direct Y-code acquisition?
4. (Carroll Alley) Computer algorithms used to infer orbital parameters and clock states. Algorithms should correct for the completely deterministic tropospheric and ionospheric delays and the relativity of simultaneity to go from measured pseudoranges to geometric ranges before activating the Kalman filter statistical estimator. The only parts of the system exhibiting statistical fluctuations are the atomic clocks, and these fluctuations are very small. The current procedures seem to be giving incorrect values of clock states and orbital parameters in the broadcast message.

**Summary**

*Problem Statement*

The well-established two-step technique for acquiring GPS P (or Y) code by transitioning from C/A code is not reliable when L1 signals are jammed by own or hostile forces. This makes it imperative to devise a means to acquire P (Y) code on L2.

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<sup>4</sup> *The Global Positioning System, A Shared National Asset, Recommendations for Technical Improvements and Enhancements*, National Academic Press, 1995. Committee on the Future of the Global Positioning System, Commission on Engineering and Technical Systems, National Research Council.

### *Background*

Code acquisition is a search process wherein GPS user equipment positions a locally generated P (Y) code in time and frequency to match (correlate) with satellite signals received at the user's antenna. Time to acquire is a function of the time delay and frequency offset to be searched, the signal-to-noise ratio of the detection observable, and the signal processing resources available in the user equipment.

### *Technology Needs*

The actions needed to satisfy mission needs for direct P (Y) code acquisition include one or more of the following:

1. Reduce the time and frequency uncertainty that must be searched.
2. Reduce the level of jamming or unintentional RFI that competes with the GPS signal in the acquisition search.
3. Increase the signal processing resources available for acquisition (e.g., multi-correlation parallel processing hardware).

### *Analysis and Recommendations*

*Item 1:* Initial acquisition uncertainty is strongly mission dependent. For example, GPS rf signals are currently inserted directly into the ATACMS and MLRS rocket launchers, so that the GPS receiver in the munitions is tracking GPS signals before launch. After the rocket clears the launch, the primary uncertainty for (re-) acquisition is acceleration-dependent (crystal) clock error and vibration errors, which are manifest by time and frequency offset and phase jitters, and missile dynamics (position and velocity). The magnitude and home variation and clock g-sensitive effects are not predictable from information available today.

For prospective gun-launched projectiles (e.g., Navy 5-in. gun, Army 155-mm gun) or airborne weapons drops, it may not be possible to initialize the weapon with GPS rf signals, and the vibration environment is extreme. However, some initialization is needed, because weapon flight times are too short for receiver operation to be achieved from a cold start. Options include time and date (and key) initialization using standard data bus interfaces (RS-422, MIL-STD-1553B) and discrete time-coded time-mark pulses. Low ground vibration sensitivity is also needed.

The committee made the following recommendations with respect to item 1.

**Recommendation 1:** Characterization and mitigation of the g-sensitivity and vibration sensitivity of crystal clocks is the highest priority goal of research to support P (Y) code acquisition for Army applications.

## *Jamming*

Recommendation 2: Key loading and data loading techniques for initializing munitions should be standardized for operation utility.

Recommendation 3: The government should maintain a watching brief on signal processing technology but not invest unless industry fails to introduce needed improvements.

*Item 2:* Improved electronic counter-countermeasures (ECCM) for GPS is a primary recommendation of the Defense Science Board (DSB) report on GPS Vulnerability and Exploitation (Delaney Committee). Their recommendations have been presented to the Office of the Vice Secretary of Defense for Acquisition (OVSD(A)); it is believed that OVSD(A) will direct the services to prioritize ECCM research within the Technology Development Plans within currently available funding.

Recommendation 4: The Army should make incorporation of increased ECCM into Army-operated user equipment a goal for its Technology Development program. This is the second highest priority recommendation.

*Item 3:* Industry investment, delivered by small infusions of contract funding from the GPS/JPO, is driving advances in parallel processing approaches to GPS signal acquisition.

Recommendation 5: The Army does not need to duplicate investment into parallel processing technology for GPS acquisition.

## **Group 6      Jamming**

Moderator: Kenneth Johnston

Secretary: Kenneth Johnston

### **Suggested Topics**

1. What are the current limitations of GPS resulting from shortcomings of the current design of user equipment (receivers and antennas), i.e., noise, multipath, jamming, etc? (By user equipment, we mean equipment that is based on current technology—equipment that is fielded and equipment that could be made based on existing technology, even if it is not yet fielded.)
2. (Tom Bahder) What are the theoretical signal processing considerations/limitations in a jamming environment?
3. (Richard Greenspan) Will we have adequate procedures for handling cryptographic key distribution for authorized GPS users? What will be the influence of planned implementation of electronic key distribution? Are the constraints imposed for protecting keyed receivers understood and acceptable to the field Army? If not, they need to be worked.

## **Summary**

The effects of jamming on GPS performance can be severe. We must assume in all military uses of GPS, especially those involving battlefield conditions, that there will always be jamming. It is further assumed that an INU (inertial navigation unit) system alone cannot be used to guide the munitions to the target. Three presentations, given by Albert Killen, Brian Baeder, and Daryl Thornburg, dealt with jamming. In summary, without protection from jammers, GPS performance for position determination can be severely limited by jammers as weak as 1 W within distances of 10–50 km of the jammer. Thus the INU system cannot be updated by GPS within this distance to the jammer. To counter this, the antenna pattern of the GPS receiver can be modified to “notch” out the jamming signal through digital beamforming and signal processing. This has been discussed by Daryl Thornburg, who has shown that digital beam forming can be accomplished with rf/LSI (large-scale integration) technology at an estimated cost of \$40K per unit. This cost does not include development costs. Therefore for different applications of this technology to GPS guided munitions, a cost/benefit ratio analysis should be made. For the MLRS, which costs only about \$20K each, the application of this technology could lead to improved accuracy, thus allowing for fewer rounds to neutralize a target and thus reduce cost significantly in the areas of transportation, number of rounds, etc. Such cost savings would result only when a very large number of MLRSs are needed to neutralize a target. For the Tactical Missile System (TACMS), which costs about \$1M, the application of this technology has a much better cost/benefit ratio. This application is being investigated for a fairly simple system, in which the GPS antenna is “notched” in the direction of the target so that the received GPS signals are (only) from overhead. Thus in all cases, cost will be an overriding factor in the application of this technology. It is estimated that a development program to apply this technology to specific military systems would cost about \$5–50M.

## **August 3: Thursday Morning Session — Time Transfer and Relativity**

### **Group 7 Relativistic Effects**

Moderator: Bill McCorkle

Secretary: Tom Bahder

#### **Suggested Topics**

1. (Bahder, Malys, Everitt) What is the source of the correlations between GPS composite clock (CC) and U.S. Naval Observatory (USNO) time?
2. (Bahder, Van Flandern) What effects become important when the autonav (satellite–satellite communication) scheme is implemented?
3. (Bahder) What relativistic effects are currently not included in the GPS, and how big are the effects?
4. (Neil Ashby) General relativity is adequate for analysis of GPS.
5. (Neil Ashby) Are understood relativity effects correctly implemented? Latest version of ICD-200 (interface control document) has correct description: frequency offset of satellite clock,  $e \sin E$  correction, Sagnac effect—in an Earth-centered Earth-fixed (EFEC) rotating frame, or propagation delay in Earth-centered inertial (ECI) frame.
6. (Carroll Alley) Identification and correction of the systematic errors. That there are major systematic errors is shown by the great success of the differential GPS procedure, in which corrections found from measured pseudo-ranges at sites of known location are broadcast in real time to users in a certain neighborhood. Other evidence is in the persistent correlations from day to day of the differences between space vehicle clocks and the master clock as measured at the U.S. Naval Observatory. If the sources of these systematic errors can be clearly identified, they can be corrected, and the intrinsic accuracy of the system for single-receiver (non-DGPS) users, which seems to be on the order of 1 m, can be achieved. This accuracy is needed for many applications. Actual accuracy in the field without DGPS seems to be no better than tens of meters.
7. (H. Yilmaz) A simple interpretation and a clear derivation of the simultaneity correction in GPS is needed so that everyone involved can understand and correctly evaluate the results. The usual interpretation via the Sagnac effect seems to be indirect, difficult to understand, and at times confusing. To achieve the desired simplicity and clarity, strict adherence to the following two statements seems to be needed: (1) physical measurement is a local process, and (2) local (measured) velocity of light is  $c$ . “Local” means a small reference frame at the observer, the frame not necessarily being inertial. The statements nevertheless allow comoving inertial frames to which the measurements can be referred. Anything nonlocal to the observer becomes his

inference. It turns out that the primary (first-order) GPS correction of concern to the receiver is simply a simultaneity effect, which is further confirmed by the actual data.

## **Summary**

The relativity session started out with comments by Yilmaz on the subject of Galilean versus Lorentzian relativity, and a commentary on the Sagnac effect. (See the Yilmaz personal statement in app B.)

Next, the discussion was led by Misner, who stressed the theoretical general relativistic principles that must be employed in GPS. (See the Misner personal statement in app B.) Misner stressed the idea that the central principle in GPS is the transformation between the proper time read on a satellite clock and the coordinate time in a (convenient) coordinate system. This transformation is given by integration of the metric along the world line of a clock. Furthermore, Misner stressed that computations in GPS should be based on physical space-time events. One space-time event is associated with the emission of a signal from the satellite, and another event is associated with the absorption of the signal by the receiver. For further details, see the personal statement by Misner in appendix B.

There was some discussion on whether GPS is fully implemented in the ECI frame coordinates. Fliegel commented that he believes that GPS is *not* fully implemented in the ECI coordinates.

There also was some discussion on how synchronization of clocks should be carried out in a general reference frame. It was generally agreed that synchronization of GPS clocks should be carried out in the sense of coordinate time synchronization.

Carroll Alley stated that it is not necessary to update the satellite clocks every 15 minutes. Furthermore, he stated that he believes that GPS should be implemented by taking the gravitational red shift effect into account by slowing down the satellite clocks (as currently implemented), but that everything else should be done using Lorentz transformation into a user's comoving frame.

There was some discussion of relativistic effects in the case of cross-link ranging (autonav). Some skepticism was expressed about whether the current approach, making piecemeal relativistic corrections, will suffice for cross-link ranging. There was some discussion of whether the calculations done by the OCS are equivalent to a clean general relativistic picture, as described by Misner. Fliegel recommended that the following action item should come out of this discussion: For cross-link ranging (autonav), we should determine whether the calculations that OCS will perform are equivalent to implementing GPS in a clean way using general relativity.

**Group 8    GPS Accuracy Issues**

Moderator: Brian Baeder  
Secretary: Albert Killen

**Suggested Topics**

1. (Paul Olson) Many Army applications using GPS and Digital Map Databases are finding out that the two do not agree in many cases. There are numerous sources of these differences, which can have devastating impact on certain applications, such as line-of-sight calculations, route planning, path guidance, targeting, etc. Developers and system users need to be cognizant of this problem.
2. (Tom Van Flandern) Can the present system achieve single-fix absolute position determinations with better than 1-m precision? Is this useful?
3. (Tom Van Flandern) What is the leading source of error in the present broadcast ephemerides? How easy would it be to eliminate it?
4. (Everett Swift) Synchronization and syntonization of monitor station clocks.
5. (Al Killen) Why can't I have a better clock? Why can we not get more correlators on a chip?
6. (Carroll Alley) Tropospheric and ionospheric parameters. The Air Force monitor stations do not monitor the local temperature, pressure, and humidity in real time and use them in the model for tropospheric delay; instead they rely on average values. Although only the pressure has a major effect, the actual values of all relevant parameters should be used. The ionospheric delay is measured with the two-frequency technique. An average value of the electron column density at the monitor stations seems to be used in the algorithm. Use of the current value could probably improve the accuracy of the calculated delay.
7. (Neil Ashby) GPS is based on a set of constants referred to as "WGS-84" (World Geodetic Survey, 1984). Better determinations of  $GM$  (where  $G$  is the universal gravitational constant and  $M$  is earth mass), quadrupole moment coefficient, average equatorial radius, etc, are now available, as in JGM-2 (the Joint Gravity Model 2 of the Goddard Space Flight Center and the University of Texas). Could slightly better performance result from updating the constants? For example, it might make a small difference in the nominal GPS satellite orbit radius. This is not a relativity issue per se, but it has an impact on the relativity. I think there may be a committee studying this; there was some discussion about it at the Performance Analysis Working Group meeting (PAWG '94).

## **Summary**

This discussion group considered areas where GPS accuracy improvements could be found. Actual system accuracy is better than specified system accuracy, probably on the order of 7 to 8 m. It is the opinion of this discussion group that further improvements can be recognized with minimum expenditures. Two areas, the space/control segment and user equipment, were considered in detail. As a result of these discussions and in light of current fiscal constraints, a prioritized list of items was developed, believed to offer the maximum cost/benefit ratio. It was determined that the space/control segment should be the first priority, since all users would benefit from these improvements. The list follows:

### *Space/Control Segment*

1. Increase the number of OCS/monitoring stations to better determine actual satellite position and performance.
2. Use single partition filter processing to take advantage of the data generated by item 1.
3. Increase the frequency of the satellite updates to maximize the benefit gained by item 2.

### *User Equipment*

1. Develop improved clocks.
2. Use all-in-view receiver technology.
3. Implement information provided in subframe four of the navigation message concerning satellite clock performance.
4. Implement host/launcher data averaging for improved ionospheric corrections, with provisions for relaying these corrections to the missile/user.
5. Implement local meteorological input for improved tropospheric corrections.
6. Improve receiver thermal/measurement noise.

It is the opinion of this discussion group that in order to maximize the utility of GPS, development of the above areas should be expedited.

**Group 9    Clocks (Part I)**

Moderator: John Vig

Secretary: Steve Jefferts

**Suggested Topics**

1. (Tom Bahder) The National Research Council (NRC) has recently completed an extensive study of GPS and has proposed many recommendations for areas of improvement.<sup>4</sup> One of the recommendations is that inexpensive quartz oscillators should replace some of the atomic clocks on some of the satellites. The NRC study recommends that a single ensemble clock should be created from the clocks on the 24 satellites, and that through autonomous ranging operations, each satellite clock offset can be corrected every 15 minutes. The NRC committee claims that such frequent clock offset correction will allow the use of less accurate (by two orders of magnitude), but more reliable quartz crystal oscillators in some fraction of the satellites. The estimated cost of three atomic clocks is approximately 3 percent of the cost of a whole satellite. Is such a trade-off shortsighted in view of future increased accuracy requirements in GPS?
2. What is the projected future accuracy and reliability of various clocks and estimated dates for achieving improved accuracies?
3. (Richard Greenspan) Is there operational doctrine for using the high-quality timing information available from GPS? (We should be able to get to a few tens of nanoseconds or better in a theater of operations.)
4. (Richard Greenspan) Can we improve our understanding of the operation of crystal oscillators under the high-g shock of gun firing? This goal relates to the intent of the Army (and Navy) to develop GPS-based "competent munitions," for which the rapid acquisition of Y-code after the round leaves the gun barrel is a very desirable feature.

**Summary**

This session concentrated mainly, but not exclusively, on the requirements of clocks in user equipment. The characteristics of a broad range of oscillators, ranging from uncompensated quartz oscillators to cesium clocks, were discussed. Several points of agreement, enumerated below, were reached by the group.

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<sup>4</sup> *The Global Positioning System, A Shared National Asset, Recommendations for Technical Improvements and Enhancements*, National Academic Press, 1995. Committee on the Future of the Global Positioning System, Commission on Engineering and Technical Systems, National Research Council.

1. An acceleration-insensitive quartz oscillator should be developed, as there are a wide range of military applications of GPS that require precise frequency control through periods of high shock and vibration.
2. Microcomputer crystal oscillators (MCXOs) show great promise for use in low-power direct P-code user equipment and should be the subject of further development.
3. Recent proposals to use only crystal oscillators as the space vehicle clocks are probably not a good idea and should be discouraged, at least until autonav is proven and the impacts of not using atomic clocks are fully determined.
4. Subcontractors for the various elements (e.g., cesium and rubidium clocks) of the space vehicles should be required to submit complete documentation of the design and construction of these items for archival purposes. The opinion was expressed that, without this requirement, the art of building satellite vehicle (SV) clocks could be lost when the present generation of scientists, engineers, and technicians who build these devices retires. Moreover, the benefits of what is learned under these subcontracts should be disseminated, so that subcontractors do not have to waste resources "reinventing the wheel."

## **August 3: Thursday Afternoon Session — Implementation Issues**

### **Group 10 *Earth Models and Orbit Determination***

Moderator: Steve Malys

Secretary: Jim O'Toole

#### **Suggested Topics**

1. (Tom Van Flandern) What can be done to improve the modeling of the eclipse season effect on satellite accelerations?
2. (Everett Swift, Steve Malys)

Earth modeling:

- Earth orientation predictions, including zonal tide effects on UT1 (Universal Time 1)
- Plate tectonic model

Force modeling:

- Tidal potential models
- Earth albedo model

3. (Jim O'Toole, Everett Swift) Fixed lag smoothing.
4. (Carroll Alley) Orbital parameters. That these can be determined sufficiently well to allow a dynamical accuracy of several tens of centimeters is shown by the post-fit analyses carried out routinely by the International Geopositioning Service at the Jet Propulsion Laboratory, the Naval Surface Warfare Center, and other places. These post-fit orbits are used to good effect in the geodetic applications of the GPS. Their accuracy has been verified to a limited extent by analyses of the laser tracking data for SV's 35 and 36. The correct analysis of the continuous tracking data from the five Air Force GPS monitor stations should allow similar accuracy for the broadcast ephemerides.

#### **Summary**

While there were no broad recommendations for further improvements in the area of "Earth modeling" that would benefit GPS orbit determination, several specific tasks were identified that could further our understanding of GPS orbit determination error budgets:

1. Perform a comparison between the OCS and Defense Mapping Agency (DMA) Earth-centered inertial (ECI) to Earth-centered Earth-fixed (ECEF) transformation, and their respective measurement model implementations.

This is valuable from the point of view of uncovering errors, interagency comparison and evaluation of products, and the proper use of products.

2. Standardize to the International Terrestrial Reference Frame (ITRF) where practical. This is valuable for implementing models and for the comparison and use of products. Note that DMA is responsible for this and has already made refinements to WGS-84 that have brought it into close coincidence with ITRF.
3. Explore tuning of the OCS filter to take into account distinct process noise regimes.

In addition to identifying the above possible tasks, the group discussed the implementation of a plate tectonic model. The consensus was that DMA should periodically (every few years — TBD) issue revised coordinates for the DoD GPS monitor stations. These revisions would be necessary, in part, because of plate motions that displace these stations up to 7 cm/year in a horizontal direction.

## **Group 11    *Clocks (Part II)***

Moderator: Gernot Winkler

Secretary: John Vig

### **Suggested Topics**

1. (Marc Weiss, Steve Jefferts) GPS works by having synchronized transmission of timing signals from known locations in space. Thus we need—

- position of SV's,
- synchronization of SV clocks.

The best way to do this is to measure these things. Unfortunately, there is no ranging of SVs from the ground, presumably for security reasons. So the whole question of how to improve GPS boils down to how best to determine these two things in real time. Our proposals:

- Use cross-link to measure the SV clocks against each other. Download this info to the control segment.
- Use two-way time transfer to link the monitor station clocks, so that the pseudorange measurements become true range measurements, plus one unknown constant: synchronized space clocks minus synchronized monitor station clocks.
- Given cross-link, focus on short-term stability in space. Rb is the likely candidate with current technology. Laser-pumped Rb is probably the best in 10 years.

## **Summary**

A clock overview was presented by Dr. Gernot Winkler, with the following main points:

Available now are the following that could improve GPS:

- Better Cs clocks for monitor stations (HP 5071's to replace HP 5061's)
- Small, inexpensive (<\$2000) Rb clocks for RAIM

Available later, after further R&D:

- Better clocks for SV—for improved autonomy
- H-masers to provide 10× to 100× lower noise than Cs standards
- Smaller, cheaper Cs clocks for RAIM
- Improved software

The group had the following additional conclusions, concerns, and recommendations:

- What are the long-term goals for GPS? Strategic clarifications are needed, for both the users and the R&D community. (Higher accuracy? Higher reliability/longer life?) Need a “good” survey of long-term user requirements. If improvements are needed, then changing the IIF specifications should be considered as soon as possible.
- On the NRC report for using crystal clocks alone in the SV: (1) relying on more frequent uploads makes the system much more vulnerable because the monitor stations are vulnerable, and (2) changing to lower quality SV clocks should be considered only after autonav is proven, and after a careful analysis of system performance with autonav as a function of clock performance.
- We need to develop “good” clock specifications for both the SV and user clocks.
- DoD needs a sustained R&D program on both SV and user equipment clocks.
- The monitor station clocks, synchronization, and software can be improved at modest cost.

**Group 12 GPS Policy, JPO Policy, and Funding of Future Work**

Moderator: Bill Howard

Secretary: Bill Klepczynski

**Suggested Topics**

1. (Paul Olson) GPS JPO has been announcing changes to the navigation messages, subframes 2/4, to improve performance to differential performance accuracies. What are the changes and to what degree will our receivers require changes to be able to take advantage of this?
2. What are the critical issues in GPS JPO policy management?
3. (Bill Klepczynski) Funding and facilitating research work:
  - (a) What studies will actually be undertaken?
  - (b) Who will fund them?
  - (c) Will Commands incorporate the issues (studies to be done) into their original requirements documents (ORDs) or some other document to bless the studies?

The last item is necessary in order to allow people to work on the issues even if they do not get formal funding.

**Summary**

1. According to technical experts at this workshop, performance improvements in GPS are possible that will take it from present levels of accuracy (about 7.5 m (3-d rms) to 3 m (3-d rms) at negligible cost on a time scale of 3 to 5 years, and to at least 2 m (3-d rms) (and possibly to 1.0–1.5 m (3-d rms)), on time scales of about 5 years. Improvements beyond 1 m will require significant time and effort and will be achieved only after significant costs are incurred.
2. The cost of these improvements, to levels of accuracy down to at least 1.5 m, is less than \$100M, which, spread over all participants and over the few years required to effect them, appears to be affordable, even within the present funding environment. The cost of these improvements amounts to a very small percentage of the total system cost, and they do not affect the satellites themselves.
3. We believe that the Services (with few exceptions) have not yet effectively articulated requirements that will permit these very significant improvements to be initiated. We urge each Service to review the future tactical and

doctrinal improvements in warfighting capabilities that these new capabilities would result in, and to establish appropriate requirements and funding priorities to bring these new capabilities to reality.

4. Improvements in the accuracy of the GPS constellation would lead to commensurate, simultaneous increases in the accuracy of differential GPS (DGPS) techniques. Many of those DGPS improvements will lead to additional capabilities in warfighting, whose requirements still require Service scrutiny and requirements articulation. The workshop participants note that, were these improvements in GPS accuracy to be made expeditiously, the Department of Defense would probably be able to avoid significant expenditures that might otherwise be made in implementing DGPS for those applications that require accuracies in the range of 1 to 5 m.

The following applications of improved GPS accuracies were noted:

1. If the accuracy of a strike involving GPS techniques is less than half the narrowest dimension of a target, we may be able to conduct many strikes without the need for additional, expensive end-game sensors. These GPS improvements to 1.5 m fulfill this condition very well, since the narrowest dimension of a tank is about 3 m.
2. Mine laying and recovery on land, at sea, and for amphibious operations, and the mapping of safe corridors can be improved in area to better than a factor of four over what current practice permits. Navigating through safe corridors will be also improved by a factor of about four.
3. We will be able to improve target acquisition time and target identification.
4. We will be able to speed up and improve the location of battlefield casualties.

The following implications of 3-m GPS accuracy were noted by workshop participants:

With 3-m accuracy, we can —

1. Get at least a fivefold improvement in weapons lethality with the same munitions payload.
2. Reduce collateral damage to the surroundings of a GPS-aided strike by a factor of 5.
3. Simplify and reduce the cost of precision-guided missiles.
4. Reduce the logistics “tail” by a factor of between two and five.

*GPS Policy, JPO Policy, and Funding of Future Work*

5. Increase the stand-off distance, improving the survivability of delivery platforms and friendly forces.
6. Increase the weapons range by reducing munitions payload, since pinpoint strikes require lower levels of payload weight.

We also note the following likely results of an improvement to 3-m accuracy:

7. Improved GPS will drive efforts for more precise target location.
8. Improved GPS will drive efforts for more accurate maps.
9. Improved GPS will minimize fratricide, because more accurate combat location will be achieved.

## **6. Conclusion**

Many complex issues were discussed at the Army GPS workshop. These issues included technical questions about the implementation of GPS, performance criteria that are being used to measure position accuracy of GPS, the outlook for future improvement in GPS position accuracy, its associated costs, and the implications for Army applications. Policy and funding issues were also discussed at the workshop in group 12. This group brought up issues that are critical to supporting continued research in this area. (The summaries of the discussions at the workshop are given in section 5.)

This workshop is recognized as only a starting point for further discussions. However, some light was shed on questions dealing with the prospects for order of magnitude improvement in GPS position accuracy and its significance to Army.

Several workshop participants provided personal statements. The goal of these statements is to clarify the participant's position on specific issues. These statements are included in appendix B.

## **Acronyms**

ATACMS	Army Tactical Missile System
C/A	coarse acquisition
CC	composite clock
CEP	circular error probable
CRPA	controlled radiation pattern antenna (null steering)
DGPS	differential GPS
DMA	Defense Mapping Agency
DSB	Defense Science Board
ECI	Earth-centered inertial
ECCM	electronic counter-countermeasures
EFEC	Earth-centered Earth-fixed
GPS	Global Positioning System
GLONASS	global navigation satellite system (Russian)
ICD-200	interface control document
IIF	(satellite designator)
IMU	inertial measurement unit
INS	inertial navigation system
INU	inertial navigation unit
ITRF	International Terrestrial Reference Frame
JGM-2	Joint Gravity Model
JPO	Joint Program Office
J/S	jammer to signal ratio
LSI	large-scale integration
MCXO	microcomputer crystal oscillator
MLRS	Multiple Launch Rocket System
MNP	Master Navigation Plan
NRC	National Research Council
OCS	operational control segment
ORD	original requirements document
OVSD(A)	Office of the Vice Secretary of Defense (Acquisition)
PAWG	Performance Analysis Working Group
P-code	precision code
PPS	precise positioning service
RAIM	receiver autonomous integrity monitoring
rf	radio frequency
RFI	radio frequency interference
SEP	spherical error probable
SIS	signal in space
SPS	standard positioning service
SV	satellite vehicle
TACMS	Tactical Missile System
UERE	user equivalent range error
URE	user range error
UT1	Universal Time 1
WGS-84	World Geodetic Survey, 1984
Y-code	encrypted P-code

# Appendix A. Position Accuracy Specifications

by Thomas B. Bahder

GPS position accuracy specification has been a source of confusion. In part this is because accuracy can be quoted in one, two, or three dimensions, and using different measures. At least four common accuracy specifications are used.

## A-1. Circular Error Probable (CEP)

When we are dealing with position accuracy specifications in two dimensions, circular error probable (CEP) is a useful measure. When GPS position accuracy is equal along two orthogonal axes that lie on the Earth's surface, it is useful to talk about a probability distribution that is circularly symmetric. By this we mean that the contours of constant probability form circles in the  $x,y$  plane, where the  $x$ -axis runs parallel (or tangent) to a line of constant latitude, and the  $y$ -axis runs parallel (or tangent) to a line of constant longitude. The accuracy termed CEP is defined as the radius of a circle centered at the peak in the distribution, such that half the probability lies inside (or outside) the circle. Note that this definition makes use of the circular symmetry of the probability distribution. For the simple case of a circularly symmetric normal distribution that peaks at  $(x_0, y_0)$  and has a standard deviation  $\sigma$  along  $x$  and along  $y$ ,

$$p(x,y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(x-x_0)^2}{2\sigma^2}\right) \exp\left(-\frac{(y-y_0)^2}{2\sigma^2}\right), \quad (\text{A-1})$$

the CEP position accuracy is defined as the radius  $R$  such that

$$\frac{1}{2} = \int_R p(x,y) dx dy, \quad (\text{A-2})$$

where the integral is done inside a circle of radius  $R$ . For the above circularly symmetric distribution,  $p(x,y)$ , the CEP value  $R$  and the standard deviation  $\sigma$  are related by

$$R = (2 \log 2)^{1/2} \sigma, \quad (\text{A-3})$$

where  $(2 \log 2)^{1/2} \approx 1.1774$ .

If a probability distribution has elliptical symmetry and is characterized by two different standard deviations along these two directions (which may not coincide with the axes used), then CEP as defined above loses its meaning. However, one may choose to define a measure of accuracy for such a distribution by doing the above integral inside an ellipse, which is a contour

## Appendix A

of constant probability, and then take as a definition of the effective radius  $R = \sqrt{ab}$ , where  $a$  and  $b$  are the lengths of the semimajor and semiminor axes of the ellipse.

### A-2. 2 drms

Position accuracy for a probability distribution in two dimensions, which is characterized by standard deviations along the  $x$  and  $y$  axes,  $\sigma_x$  and  $\sigma_y$ , is often given in terms of the 2 drms value, which is defined by

$$2(\sigma_x^2 + \sigma_y^2)^{1/2} . \quad (\text{A-4})$$

Note that this measure of position accuracy characterizes a two-dimensional probability distribution by one number. For the circular distribution given in equation (A-1), the 2 drms position accuracy is

$$2\sqrt{2}\sigma \approx 2.8284\sigma . \quad (\text{A-5})$$

Consider two circularly symmetric distributions, where one distribution is characterized by 2 drms position error and the other by CEP. If the 2 drms position error is equal to the CEP position error, then the standard deviation of the distribution characterized by the 2 drms criteria,  $\sigma_{2\text{drms}}$ , is related to the standard deviation of the distribution characterized by CEP,  $\sigma_{\text{CEP}}$ , by

$$\sigma_{2\text{drms}} \approx 0.41627\sigma_{\text{CEP}} .$$

Note that this means that 2 drms is a more stringent characterization of error than CEP.

### A-3. Spherical Error Probable (SEP)

For a three-dimensional accuracy specification, when the probability distribution  $p(x,y,z)$  is spherically symmetric, we define the spherical error probable (SEP) as the radius  $R$  of a sphere such that half the probability lies inside the sphere by

$$\frac{1}{2} = \int_R p(x,y,z) dx dy dz , \quad (\text{A-6})$$

where the  $z$ -axis is orthogonal to  $x$  and  $y$ , and the integral is done over the volume of a three-dimensional sphere of radius  $R$ . For example, consider a spherically symmetric Gaussian probability distribution centered at  $(x_0, y_0, z_0)$  with standard deviation  $\sigma$  in  $x$ ,  $y$ , and  $z$ :

$$p(x,y,z) = \frac{1}{(2\pi)^{3/2}\sigma^3} \exp\left(-\frac{(x-x_0)^2}{2\sigma^2}\right) \exp\left(-\frac{(y-y_0)^2}{2\sigma^2}\right) \exp\left(-\frac{(z-z_0)^2}{2\sigma^2}\right) . \quad (\text{A-7})$$

Carrying out the integral in equation (A-6) leads to

$$\frac{1}{2} + \frac{2}{\sqrt{\pi}} xe^{-x^2} - \operatorname{erf}(x) = 0 \quad (\text{A-8})$$

where  $x = R/\sqrt{2}\sigma$  and

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt . \quad (\text{A-9})$$

Numerical solution of equation (A-8) gives the approximate relation between the CEP value,  $R$ , and the standard deviation,  $\sigma$ ,

$$R \cong 1.538 \sigma . \quad (\text{A-10})$$

Obviously, if the distribution has the symmetry of a spheroid, then the integral can be done over a spheroid of the same symmetry, and an effective radius can be defined.

#### A-4. 3-d rms

We can express three-dimensional accuracy operationally by computing the standard deviation in distance between successive measurements of position of a known position point. For example, we take  $N$  data points consisting of position vectors  $\mathbf{r}_n = (x_n, y_n, z_n)$ , where the Cartesian coordinates of the point are  $(x_n, y_n, z_n)$  for measurement  $n$ . If the known position is given by a vector  $\mathbf{r}_0$ , the 3-d rms error in distance is given in the usual way as

$$\sigma = \left[ \frac{1}{N} \sum_{n=1}^N |\mathbf{r}_n - \mathbf{r}_0|^2 \right]^{1/2} \quad (\text{A-11})$$

If the vectors involved are two-dimensional, an analogous position accuracy specification can be defined for the case of a 2-d rms error. When a measurement is made of the distance to a satellite (rather than the coordinate position), the error can be called 1-d rms or user range error (URE). Note that URE can be distinct from user equivalent range error (UERE). Specifically, UERE is a term reserved for the error in the GPS user equipment set's measurement of the *pseudorange* to a given satellite.<sup>1</sup>

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<sup>1</sup> GPS Navstar, Global Positioning System: User's Overview (prepared by ARINC Research Corp. for the Program Director, Navstar Global Positioning System Joint Program Office), YEE-82-009D, March 1991.

## **Appendix B. Participants' Personal Statements**

The following pages are reproductions of personal statements that were submitted by workshop participants. These statements are meant to convey in more detail the point of view of the given participant.

<i>Some Comments, Conclusions and Recommendations Relating to the Army GPS Workshop, Chapel Hill, NC, 2–3 August 1995 .....</i>	39
by Carroll O. Alley, University of Maryland at College Park	
<i>Relativity Issues in the GPS .....</i>	43
by Neil Ashby, University of Colorado	
<i>Precis of General Relativity .....</i>	53
by Charles W. Misner	
<i>GPS and the Local Velocity of Light .....</i>	59
by Hüseyin Yilmaz, Hamamatsu Photonics K.K. and Tufts University	
<i>The Problem of the Unity of Physics .....</i>	65
by Hüseyin Yilmaz, excerpt from <i>Perception and Philosophy of Science</i> , vol XIII in <i>Logical and Epistemological Studies in Contemporary Physics</i> , Robert S. Cohen and Marx W. Wartofsky, editors, D. Reidel Publishing Company (1974)	
Reprinted by permission of Kluwer Academic Publishers.	
<i>Comments on GPS Workshop .....</i>	69
by Gernot M. R. Winkler	

**Some Comments, Conclusions and Recommendations Relating to the  
Army GPS Workshop, Chapel Hill, NC, 2-3 August 1995**

by

**Carroll O. Alley, Professor of Physics  
University of Maryland at College Park**

**1. Clock States**

Our data shows that a single assignment of rate and offset for each clock in the system, including monitor station clocks, works well for at least 33 hours without need for change. This point was not sufficiently emphasized in our presentation. The Kalman filter used in the OCS algorithms, even with long time constants reflected in Q values, is not averaging over a sufficiently long time interval to match the intrinsic stability of the atomic clocks. The clocks themselves are very likely not the dominant source of signal-in-space error as asserted by Aerospace. Instead, it is probably the incorrect assignment of the clock states in the process of estimating them at 15 minute intervals which makes it only appear to be the dominant error source in the Aerospace analysis. There is also the possible contribution to error in clock state estimation from the confusing iteration procedure in the OCS algorithms described in section 3. below. It is recommended that the proposed WAGE procedure of more frequent upload of clock states be carefully reconsidered before implementation. It may make the errors larger.

**2. Diagnostic Measurements and Analysis of Data.**

The results on accuracy presented by Aerospace were deduced by unclear statistical procedures whose measurement data base was also unclear. The Kalman filter process seems certainly to be involved in some way. Our data analysis, on the other hand, is very simple and straightforward. It uses raw pseudorange measurements (smoothed over only 6 seconds) which are compared every 15 minutes by subtracting the geometric range calculated from the "true" post-fit orbits provided by the International GPS Service (IGS) at the Jet Propulsion Laboratory. The residuals, after correcting for tropospheric and ionospheric delays and the relativity of simultaneity, are exhibited as a function of time for horizon to horizon (6 degrees to 6 degrees) passes for each of 19 satellites and 5 monitor stations over 33 hours. The residuals are very flat with values near zero for each space vehicle / monitor station pass. They were characterized by calculating formally the standard deviation about zero of the entire data set used. The value of 2.5 meters arrived at in this way cannot be directly compared to the Aerospace data. Their procedures need to be clarified and assessed before their results can be accepted at face value. More controlled observations and calibration measurements, as discussed in my Issues statements, must be carried out, particularly for rapidly moving platforms where the relativity of simultaneity requires large corrections, before we can know for sure how well, compared with its potential, the GPS is actually performing in the field for PPS users.

**3. Relativity.**

The transformation between the Earth Centered Inertial frame (**ECI**) and the Earth Centered Earth Fixed frame (**ECEF**) has gotten mixed up with the "Sagnac Effect" Earth rotation interpretation of the relativity of simultaneity espoused by Ashby. His reason for introducing this incorrect relativistic physics seems to be the inability of general relativity to

accommodate the needed special relativistic Lorentz transformations to a succession of co-moving inertial frames instantaneously at rest with respect to the receiver. It is, of course, in the ECEF where users need to find their location and where the pseudorange is actually measured. The basic premise of the GPS is to have all clocks synchronized and "syntonized" to keep coordinate time in the ECI. It is in this frame that the orbital calculations are carried out using Newtonian dynamics. Thus the transformation between the coordinates of events must be treated very carefully. The geometric transformation, which (to the accuracy needed in the GPS) does not require a relativistic treatment, must not be confused with the transformation of the measured pseudorange, which does require the recognition of the relativity of simultaneity. Pseudorange is the shorthand descriptive name given to the explicit measurement, performed locally by the receiver, of the difference between the reception time and the emission time of events labeled by their epochs in the pseudorandom time code bit stream transmitted by the space vehicles and compared with the same (shifted) time code in the receiver. The relativity of simultaneity is expressed by the Lorentz time transformation giving the epoch time of the distant emission event which must be assigned in the co-moving frame of the receiver. It is necessary to do so in order that the local speed of the electromagnetic signal measured by the receiver be  $c$  in every direction.

This requirement is not automatically contained in GR as is clear from the analysis of the Langevin metric which leads to a paradox between the Sagnac and Michelson-Morley experiments as discussed by H. Yilmaz in his comments at the workshop and in his paper distributed there, "GPS and the Velocity of Light". This paradox is resolved only by an additional statement that the metric explicitly contains the position of the observer and that locally (that is in the vicinity of the observer) the velocity of light in vacuum is  $c$  in all directions. In other words, it is not enough to just write down a general metric, as was done by C. W. Misner during the discussion, and hope that the dumb instrument will follow the wishes of the theoretician (the Langevin metric is a counter example). One must explicitly arrange that the measurement procedure of special relativity is imposed at the immediate vicinity of the observer making the actual measurement. This was explicitly shown by H. Yilmaz at the workshop

That N. Ashby misunderstands the point here involved is clear from his Eq.(35) on page 18 and the subsequent discussion in his paper, "Relativistic Effects in the Global Positioning System," distributed at the workshop.

$$c\Delta t = \left| \vec{r}_B + \vec{\Omega}_e \times \vec{r}_B \Delta t - \vec{r}_A \right|.$$

This is equivalent to having the speed of light depend on the velocity of the receiver as can be seen easily for the special case in which all the vectors are along the same line. Writing  $\vec{r}_B - \vec{r}_A = \vec{l}$ , and  $\vec{\Omega}_e \times \vec{r}_B = \vec{v}$ , we then have  $c\Delta t = |\vec{l}| \pm v\Delta t$  and

$$\Delta t = \frac{|\vec{l}|}{c \mp v}.$$

That is, the velocity of light appears as  $c \mp v$ , and not  $c$ . This is contrary to the statement that the local velocity of light should be  $c$  independently of the local velocity. Thus Ashby's analysis cannot refer to the observer on the Earth holding the receiver. It may refer to an observer at the center of the Earth in the sense of an inference (not measurement) of what is going on at the surface, but this is an inference. The observer cannot be at the center of the Earth and measure the local velocity of light at the surface at the same time.

Besides, there is no real observer at the center of the Earth. The result of Ashby refers to an inference of a fictitious non-existent observer.

In Ashby's subsequent discussion, his Eq. (18) is solved approximately for  $\Delta t$  by iteration, leading to a result which mimics to first order in  $v$  the relativity of simultaneity term in the Lorentz transformation equation for time. The Lorentz transformation is then misinterpreted as the correction to the transit time of the signal due to the motion of the receiver during the transit time, rather than correctly as the relabeling of the epoch time of the distant event as described above. Further, an incorrect local interpretation of the Sagnac effect, which actually relates only to non-local closed paths on the rotating Earth, is attached to this result. Why not avoid all this by using the Lorentz time transformation from the beginning? The answer seems to be that the metric of general relativity does not allow a local correspondence to special relativity for the actual receivers on the rotating Earth, as discussed above.

The non-relativistic notion of corrections due to the additional distance traveled by an observer on the rotating Earth during the transit time of the signal from the satellite has been incorporated in the algorithms in the Kalman filter procedure used by the OCS to process the measured pseudorange data from the monitor stations. The algorithms involve an iteration of an infinitesimal rotation combined with the main rotational transformation between the ECI and the ECEF. The tropospheric and ionospheric delay corrections are added to the so-called free-space propagation delay (the geometric range in the ECI divided by  $c$ ) in this iteration procedure. Thus, in addition to the mixing up of geometric and relativistic transformations, the errors associated with the atmospheric delays are also preventing a clean application of the relativity of simultaneity correction. This is clearly being mimicked to some extent, but this whole confusing procedure needs to be straightened out. (A nearly impenetrable account, but the only one seemingly available, is given in Appendix A to the Computer Program Development Specification for the Master Control Station Ephemeris/Clock Computer Program of the Navstar GPS Operational Control Segment produced by the IBM Federal Systems Division in 1988.) It is probably also a major source of the suspected incorrect clock state assignments discussed in section 1. above. In other error analyses of the GPS, for example that reported by Steve Malys at the workshop, the so-called Earth rotation rate correction which mimics with the wrong physics the relativity of simultaneity, is more cleanly applied.

#### 4. Cross-link Ranging.

The procedure given by N. Ashby is exceedingly complicated and confusing. Our approach requires only a simple Lorentz transformation for time which is readily evaluated and is easily understood. The magnitude of the relativity of simultaneity term will be an order of magnitude larger than for the stationary receivers on Earth because of the larger velocity of the space vehicles (3800 m/s compared with  $\approx 350$  m/s). We are attempting to acquire papers on the planned procedures in order to assess them in detail. If it is intended to reproduce the process now used in the OCS, there will be even greater problems in the mixing up of clock states and orbital parameters.

#### 5. General Conclusion

In all considerations and analyses concerning the Global Positioning System one must keep constantly in mind that it is a time based system. Its very existence rests on the remarkable stability of modern atomic clocks. All information is obtained from measurements of time. A major focus for improvements therefore should be the use of the very best atomic clocks in the monitor stations as well as in the space vehicles, and the explicit application of the correct relativistic physics in the interpretation of all measurements in the system.

**RELATIVITY ISSUES IN THE GPS\***

by

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**Abstract.** These notes summarize how relativistic effects are treated in the Global Positioning System (GPS), and provide a critical analysis of the issues raised by proposals to consider relativistic effects within a 'user' frame of reference.

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\* For the Army GPS workshop in Chapel Hill, NC, August 2-3, 1995.

## RELATIVITY ISSUES IN THE GPS

## SUMMARY

- 1. All computations can be done in the Earth-Centered Inertial (ECI) reference frame.
- 2. Relativistic corrections for frequency shifts due to motion of SV clocks, and due to their locations in the Earth's gravitational field, are all correctly included in the current system design.
  - 2.a The hardware frequency offset adjusts the SV clocks as if the orbits were circular;
  - 2.b It is the responsibility of each receiver to apply a correction arising from SV orbital eccentricity.
- 3. Effects on the user can also be computed in the ECI frame.
  - 3.a Relativistic effects due to the finiteness of the speed of light are included in the ECI frame by accounting for the user's change in position during the time of signal propagation.
  - 3.b Clocks near but not on Earth's geoid experience gravitational frequency shifts; this effect is well-understood.
- 4. At the present time (August 1995), there are no significant relativity effects, which when analyzed in or reduced to ECI coordinates, are not accounted for in existing theoretical descriptions of the GPS system.
- 5. For the Global Positioning System the ECI frame is conceptually by far the most simple and convenient to use. *All suggestions that significant relativity effects have been omitted from consideration in the GPS should be presented in ECI coordinates, as well as in other coordinate systems of choice.*
- 6. Using signals from four GPS satellites, which have satellite clock transmit time and transmit position in their messages, a receiver can determine its position, and its GPS time, by iteratively solving four propagation delay equations based on the constancy of the speed of light in ECI coordinates, appropriately corrected for refractive effects.
- 7. If a receiver is at a previously known location, fixed on the rotating Earth, then GPS time may be transferred to the receiver from a single satellite by computing the ECI range at the instant of transmission and applying a 'Sagnac' correction for receiver motion due to Earth rotation; this is equivalent to solving one propagation delay equation iteratively, but stopping after one iteration.
- 8. The Sagnac effect, observed in a rotating Earth-fixed frame, can be viewed in many equivalent ways; for example it can be conceived as arising from the relativity of simultaneity, which occurs when comparing clocks in the ECI frame with clocks in a sequence of local inertial frames, instantaneously comoving with some user.
- 9. Comoving 'user' frames unnecessarily introduce a host of irrelevant effects because such frames accelerate and rotate.

**RELATIVITY ISSUES IN THE GPS**

- 1. All computations can be done in the Earth-Centered Inertial (ECI) reference frame.

GPS time is a self-consistent coordinate time with MCS and satellite clocks rate-adjusted to agree with the SI second (referenced to USNO) on the geoid of the rotating Earth, and synchronized (initialized) in the ECI reference frame. Relativity corrections are applied to elapsed proper time on every satellite clock, to produce a coordinate time which agrees at every instant with a hypothetical coordinate clock at rest in the ECI frame, with which the satellite clock instantaneously coincides.

- 2. Relativistic corrections for frequency shifts due to motion of SV clocks, and due to their locations in the Earth's gravitational field, are all correctly included in the current system design.

- 2.a The hardware frequency offset adjusts the SV clocks as if the orbits were circular.

Accounted for in the frequency offset are: the gravitational frequency shift of satellite clocks due to Earth's monopole potential; the gravitational frequency shift of Earth-bound clocks, including Earth's monopole and quadrupole potentials; second-order Doppler shifts of satellite clocks; and second-order Doppler shifts of Earth-bound clocks due to Earth rotation.

- 2.b It is the responsibility of each receiver to apply a correction arising from SV orbital eccentricity. This effect is a combination of gravitational frequency shift and second-order Doppler shift of the Satellite clock which varies with orbital altitude and speed.

- 3. Effects on the user can also be computed in the ECI frame.

- 3.a Relativistic effects due to the finiteness of the speed of light are included in the ECI frame by accounting for the user's change in position during the time of signal propagation.

- 3.b Clocks near but not on Earth's geoid experience gravitational frequency shifts; this effect is well-understood.

- 4. At the present time (August 1995), there are no significant relativity effects, which when analyzed in or reduced to ECI coordinates, are not accounted for in existing theoretical descriptions of the GPS system.

The most significant relativity effect which has not yet been incorporated appears to be a gravitational frequency shift of the atomic clocks in GPS satellites, due to the Earth's quadrupole moment. This effect has a six-hour period and an amplitude equivalent to about 4 cm of delay. Effects due to the user's ground motion can also be computed in the ECI frame; motional second-order Doppler shifts due to user ground speed contribute less than 100 ps for speeds less than 20 km/s, during the propagation time of a signal.

It is possible that some software may not have fully implemented already well understood relativity concepts. To reduce controversy it would be desirable for receiver manufacturers to document the manner in which relativity corrections are implemented.

- 5. For the Global Positioning System the ECI frame is conceptually by far the most simple and convenient to use.

One of the tenets of General Relativity is that all reference frames are equivalent as regards

the description of physical phenomena. The success of the GPS, the simplicity of treatment in ECI coordinates, and the extensive existing analysis in ECI coordinates leads to the following proposal:

*All suggestions that significant relativity effects have been omitted from consideration in the GPS should be presented in ECI coordinates, as well as in other coordinate systems of choice.*

- 6. Using signals from four GPS satellites, which have satellite clock transmit time and transmit position in their messages, a receiver can determine its position, and its GPS time, by iteratively solving four propagation delay equations based on the constancy of the speed of light in ECI coordinates, appropriately corrected for refractive effects.

This will give ECI position and GPS time at the receiver; a simple rotation will then give Earth-fixed position. The solution of the propagation delay equations can proceed by iteration, without any need to consider Lorentz transformations to a user frame of reference, breakdown of simultaneity, or any other relativity effects in an Earth-fixed frame. A numerical example of this is given in Appendix I of these notes.

- 7. If a receiver is at a well-surveyed or previously known location (for example, at a fiducial location at NIST), fixed on the rotating Earth, then GPS time may be transferred to the receiver from a single satellite as follows: Let  $\mathbf{r}_T$  and  $\mathbf{r}_R$  be the positions in ECI coordinates of the transmitter and receiver at the instant of transmit, and let  $L = |\mathbf{r}_T - \mathbf{r}_R|$  be the initial distance between them. Then the total light time delay is

$$t = \frac{L}{c} + \frac{2\Omega_e \cdot \mathbf{A}}{c^2} \quad (1)$$

where  $\mathbf{A}$  is the vector area swept out by a vector from the center of the Earth to the signal pulse as it travels from transmitter to receiver:  $\mathbf{A} = \frac{1}{2}\mathbf{r}_T \times \mathbf{r}_R$ , and where  $\Omega_e$  is Earth's angular rotational velocity. This correction is sometimes called a "Sagnac Correction."

Eq. 1 for the Sagnac correction is equivalent to solving one light propagation delay equation iteratively, but stopping after one iteration. This is also equivalent to accounting for the motion of the receiver in the ECI frame, during propagation of the signal from transmitter to receiver. This is already well known, but for ease of reference is proved in Appendix II of these notes.

- 8. The Sagnac effect, observed in a rotating Earth-fixed frame, can be viewed in many equivalent ways; for example it can be conceived as arising from the relativity of simultaneity,<sup>2</sup> which occurs when comparing clocks in the ECI frame with clocks in a sequence of local inertial frames, instantaneously comoving with some user.

See Appendix III. This is already well-known. The Sagnac effect prevents the construction of a self-consistently synchronized coordinate clock network in a rotating reference frame, using Einstein synchronization (constancy of  $c$ ) or slowly moving portable clocks. Two Earth-fixed users at different positions, who attempt to extend a synchronized clock network from their respective positions by such processes, will in general disagree with each other. The Sagnac correction, Eq. (1), is similar to, but differs in sign, from the Sagnac effect which would be observed using Earth-fixed rotating coordinates and synchronizing clocks using constancy of the speed of light in a local 'user' Earth-fixed frame. The reason for the sign difference is, that a correction for the Sagnac

effect must be applied to remove it, in order to obtain GPS time which is self-consistently defined only in the ECI frame. This is proved in Appendix III.

- 9. Such ‘user’ frames unnecessarily introduce a host of irrelevant effects because such frames accelerate and rotate.

Many relativistic motional and gravitational effects arise in reference frames which are leaping and whirling about in the ECI frame. One should not, for example, try to calculate the dynamics of GPS satellite motion in the rotating frame where there are fictitious Coriolis and centripetal forces. Usually the discussion of effects in such frames is very confusing; apparent paradoxes frequently arise; light does not propagate uniformly in straight lines; in some cases pairs of effects arise which cancel out and it is difficult to ferret out both members of the pair. When expressed in ECI coordinates, to date all such proposed effects have reduced to previously known effects or have disappeared. For example, it could be said that to a single user, moving with velocity  $\mathbf{v}$  relative to the ECI frame, the GPS satellite clocks will not appear synchronized. This might be relevant if the user possessed a system of synchronized clocks, at rest in the user’s frame but at the position of the GPS satellites, and with which direct observations of GPS satellite clocks could be made, but no such user clocks exist. The user has only the clock in the user’s receiver. Attempts to compute simultaneity corrections in every user frame would require that the position and velocity of the user already be known, and at the very least would lead to much wasted effort.

One justification for the introduction of ‘user’ reference frames seems to be that “there aren’t any observers in the ECI frame.” However, receivers could be put at the north and south poles, on rotating platforms with 24-hour periods; these would provide ECI observations. More significantly, each satellite or ground clock which has been set in accord with paragraph 1 above, reads ECI (or GPS) time. Instruments can provide observations without the actual presence of a person.

## Appendix I

## SOLUTION OF PROPAGATION DELAY EQUATIONS BY ITERATION

In this Appendix a numerical example of the solution of four propagation delay equations, by iteration, is given.

Consider several GPS satellites which at GPS times  $t_1, t_2, t_3, t_4$ , send out signals from locations  $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4$ , respectively, in ECI coordinates. Let these four signals be received at some instant  $t$  by a receiver at position  $\mathbf{r}$  (also in ECI coordinates). The problem is to determine the ECI time  $t$  and position  $\mathbf{r}$  at the receiver. In the present case it is not necessary to consider motion of the receiver during the time of propagation of the signal; the final position  $\mathbf{r}$  at the instant of reception  $t$  will be determined by the solution of four equations which express the condition that the speed of propagation of the signals is  $c$ , the speed of light. The four equations are:

$$(\mathbf{r} - \mathbf{r}_j)^2 = c^2(t - t_j)^2, \quad j = 1, 2, 3, 4. \quad (I.1)$$

Here boldface denotes a vector with three spatial components, and the subscripts label the information received from the different satellites. Such equations are rather clumsy to solve in general. However, if the position  $\mathbf{r}$  and time  $t$  are known approximately, the equations can be reduced to a system of four linear equations which can be solved by standard matrix inversion techniques.

Suppose that for user position and time we write:

$$\mathbf{r} = \mathbf{r}^{(i)} + \delta\mathbf{r}, \quad t = t^{(i)} + \delta t, \quad (I.2)$$

where the quantities  $\mathbf{r}^{(i)}, t^{(i)}$  are guesses or trial values for the position and time, and  $\delta\mathbf{r}, \delta t$  are assumed to be small quantities which are to be determined by solving Eqs. (I.1). The index  $(i)$  in Eq. (I.2) labels the  $i^{th}$  trial value in a repeated or iterative process of solving Eqs. (I.1).

Substituting Eqs. (I.2) into (I.1), expanding and neglecting squares of small quantities  $(\delta\mathbf{r})^2$  and  $(\delta t)^2$ , the equations can be put into the following form, for  $j = 1, 2, 3$ , or 4:

$$\delta\mathbf{r} \cdot (\mathbf{r}^{(i)} - \mathbf{r}_j) - \delta t c^2(t^{(i)} - t_j) = \frac{1}{2} [c^2(t^{(i)} - t_j)^2 - (\mathbf{r}^{(i)} - \mathbf{r}_j)^2]. \quad (I.3)$$

In Eq. (I.3), the quantity  $c(t^{(i)} - t_j)$  is the pseudorange from the receiver to the  $j^{th}$  satellite. The inhomogeneous term on the right-hand side of Eq. (I.3) is half the difference between the squares of this pseudorange and the estimated range,  $(\mathbf{r}^{(i)} - \mathbf{r}_j)^2$ .

These equations form a system of inhomogeneous linear equations which can be solved by matrix inversion. The matrix of coefficients of the unknowns  $\delta\mathbf{r}, \delta t$  on the left will usually be nonsingular, unless the configuration of satellites is so unfavorable that the solution is not unique. (It is possible for this to occur.) The solutions obtained will be approximate, but can be used to obtain new trial values; the process of solution can then be repeated. Thus, suppose at stage  $i$  in the iteration process, Eqs. (I.3) have been solved for corrections  $\delta\mathbf{r}$  and  $\delta t$ ; then better approximations can be found by setting new trial values as follows:

$$\mathbf{r}^{(i+1)} = \mathbf{r}^{(i)} + \delta\mathbf{r}, \quad t^{(i+1)} = t^{(i)} + \delta t. \quad (I.4)$$

The iteration process converges fairly rapidly, particularly if the initial trial guesses are good. The solutions will be found at some stage  $i = i_{final}$  when the inhomogeneous term on the right side of Eq. (I.3) becomes negligibly small. Then the solutions satisfy:

$$(\mathbf{r}^{(i_{final})} - \mathbf{r}_j)^2 = c^2(t^{(i_{final})} - t_j)^2, \quad j = 1, 2, 3, 4$$

## Appendix B

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which are equivalent to Eqs. (I.1).

Summarizing, the computation scheme proceeds as follows:

Step 1: Choose approximate values for user position and time.

Step 2: Solve Eqs. (I.3) by matrix inversion to obtain corrections  $\delta r$  and  $\delta t$ .

Step 3: Test whether the corrections are sufficiently small that computation can be halted; if not, combine the corrections obtained in Step 2 with the chosen approximate values of position and time to obtain new approximations for position and time, as in Eq. (I.4). Then go to Step 2.

To illustrate this process, let the positions of four satellites and the times at which signals are transmitted be as given in Table I.

Table I. Satellite positions in ECI coordinates and GPS signal transmission times.

Satellite #	x-position (m)	y-position (m)	z-position (m)	times $t_i$ (sec)
1	23 275 990.00	7 477 20.00	-10 380 062.00	-0.081 766 311 70
2	22 061 206.00	14 737 903.00	1 398 571.00	-0.075 594 689 70
3	23 760 596.00	11 786 730.00	-1 294 277.00	-0.076 767 537 70
4	10 950 172.00	16 193 927.00	17 983 056.00	-0.074 507 910 70

This example has been chosen so that the correct user time is zero; this way convergence to the correct time can easily be examined. The initial user clock reading is assumed to be -.01442357000 seconds upon reception of the four signals, while nothing is known initially about the receiver's position. For simplicity I shall assume that the initial trial values for the position place the receiver at Earth's center. Calculations are performed to more significant figures than necessary so that round-off error is not an issue. The iterative procedure then gives the following sequence of results.

Table II. User position coordinates and clock times determined by iteration.

Trial #	user x (m)	user y (m)	user z (m)	user clock time (sec)
0	0.00	0.00	0.00	-0.014 423 570 00
1	4779641.36	208250.50	4013745.90	-0.001 573 214 75
2	4876673.81	212432.68	4095899.80	0.000 013 565 76
3	4875844.32	212396.92	4095197.60	0.000 000 000 99
4	4875844.26	212396.92	4095197.55	0.000 000 000 00
5 (final)	4875844.26	212396.92	4095197.55	0.000 000 000 00

There is no further improvement after the fourth iteration; in fact three iterations are sufficient in this example. The actual ECI receiver position and time are in the last row of Table II; these coordinates will be denoted by  $(x[5], y[5], z[5])$  in the discussion.

The spacetime receiver position has thus been determined in ECI coordinates. This example shows that is not necessary to think about receiver motion, Lorentz transformations, or the Sagnac effect. All that is important is the constancy of the speed of light in the ECI frame, and accurate information about the positions and times of transmission (again in ECI coordinates) of signals from the four satellites.

## Appendix II

**EQUIVALENCE OF SAGNAC CORRECTION TO A SINGLE ITERATION  
OF A SINGLE PROPAGATION DELAY EQUATION**

In this Appendix I show how one iteration of one propagation delay equation leads to the Sagnac correction. A numerical example based on the data of Appendix I is also given.

Let  $\mathbf{r}_T$  and  $\mathbf{r}_R$  be the positions in ECI coordinates of the transmitter and receiver at the instant of transmit, and let  $L = |\mathbf{r}_T - \mathbf{r}_R|$  be the initial distance between them. Let the velocity of the user at the instant of transmit be  $\mathbf{v}$ . At the instant of reception, the position of the user will be approximately  $\mathbf{r}_R + \mathbf{v}t$ , where  $t$  is the total light time delay. (User acceleration of as much as 10 g's will cause only a few cm error in this discussion.) The delay  $t$  is determined by the constancy of the speed of light:

$$c^2 t^2 = [\mathbf{r}_T - (\mathbf{r}_R + \mathbf{v}t)]^2. \quad (II.1)$$

Expanding the right-hand side and taking a square root,

$$ct = \sqrt{(\mathbf{r}_T - \mathbf{r}_R)^2 - 2(\mathbf{r}_T - \mathbf{r}_R) \cdot \mathbf{v}t + v^2 t^2}. \quad (II.2)$$

The initial stage of the iteration is to neglect receiver motion entirely. Setting  $\mathbf{v} = 0$  in Eq. (II.2) gives for the initial estimate of  $t$  the value

$$t^{(0)} = \frac{|\mathbf{r}_T - \mathbf{r}_R|}{c} = \frac{L}{c}. \quad (II.3)$$

For a better approximation, substitute this estimate of the propagation delay into the right-hand side of Eq. (II.2) and expand the square root, keeping only linear terms in  $\mathbf{v}$ .

$$\begin{aligned} ct &\simeq \sqrt{L^2 - 2(\mathbf{r}_T - \mathbf{r}_R) \cdot \mathbf{v}L/c} \\ &= L\sqrt{1 - 2(\mathbf{r}_T - \mathbf{r}_R) \cdot \mathbf{v}/(Lc)} \\ &\simeq L[1 - (\mathbf{r}_T - \mathbf{r}_R) \cdot \mathbf{v}/(Lc)] \end{aligned}$$

so the next approximation is

$$t^{(1)} = \frac{L}{c} - \frac{(\mathbf{r}_T - \mathbf{r}_R) \cdot \mathbf{v}}{c^2}. \quad (II.4)$$

Thus, accounting in first order for receiver motion through the ECI frame during signal propagation results in a general expression linear in the receiver velocity  $\mathbf{v}$ . Eq. (II.4) would be suitable for use by Low-Earth-Orbiting satellites or high-speed jet aircraft. If, however, the receiver is fixed on the rotating Earth, this velocity is determined by the angular rotation velocity  $\boldsymbol{\Omega}_e$  of Earth,

$$\mathbf{v} = \boldsymbol{\Omega}_e \times \mathbf{r}_R. \quad (II.5)$$

Substituting this expression for  $\mathbf{v}$  into Eq. (II.4), and using the fact that  $\mathbf{v}$  is perpendicular to  $\mathbf{r}_R$  gives

$$t^{(1)} = \frac{L}{c} + \frac{2\boldsymbol{\Omega}_e \cdot \frac{1}{2}(\mathbf{r}_T \times \mathbf{r}_R)}{c^2}, \quad (II.6)$$

which is Eq. (1) of paragraph 7 above.

**A Numerical Example.** Consider the transfer of time from Satellite #1 to the receiver in the example discussed in Appendix I. To apply Eq. (II.6) the position  $\mathbf{r}_R$  of the receiver must be known at the instant of emission,  $t_1$ , whereas the result of the iteration process gives the receiver position at the instant of reception.

The initial receiver position can be found from the final position for this example by applying a rotation in the sense opposite to Earth's spin, as follows. For convenience let  $x[5]$ ,  $y[5]$ ,  $z[5]$  denote the final ECI receiver position coordinates, given in the last row of Table II. Denote the initial receiver position (which will be different for each transmit signal) by  $x_R$ ,  $y_R$ ,  $z_R$ . Then

$$\begin{aligned}x_R &= x[5] \cos(-\Omega_e t_1) + y[5] \sin(-\Omega_e t_1) \\y_R &= -x[5] \sin(-\Omega_e t_1) + y[5] \cos(-\Omega_e t_1) \\z_R &= z[5]\end{aligned}$$

(In actual practice this position would be found from the known receiver position on Earth, and the time transmitted by the satellite.) Then for Satellite # 1, and with  $\Omega_e = 7.2921151467 \times 10^{-5}$  rad/sec, the initial receiver position is

$$(x_R, y_R, z_R) = (4 875 845.53, 212 367.85, 4 095 195.55) \text{ m}$$

and the initial estimate of range is

$$|\mathbf{r}_T - \mathbf{r}_R| = L = 24 512 931.23 \text{ m},$$

which gives for the initial trial value of the propagation delay to the receiver

$$t^{(0)} = \frac{L}{c} = +0.081 766 337 27 \text{ sec.}$$

Let  $x_1, y_1, z_1$  be the coordinates at transmit time of satellite #1. These are given in Table I. Then the equatorial projection of the area swept out by a vector from the center of the Earth to the light pulse as it propagates from satellite to receiver, is

$$\frac{1}{2}(x_1 y_R - y_1 x_R) = -1.57586 \times 10^{13} \text{ m}^2.$$

Since  $2\Omega_e/c^2 = 1.62271 \times 10^{-21}$  sec/m<sup>2</sup>, from Eq. (II.6) the Sagnac correction is

$$t^{(1)} = -25.57 \times 10^{-9} \text{ sec.}$$

The final user clock time for this example is

$$t_1 + t^{(0)} + t^{(1)} = 0.000 000 000 00 \text{ sec.}$$

This example shows that for receivers with already known positions, time can be transferred from satellite to receiver by solving a single propagation delay iteratively and stopping after one iteration, or equivalently, by applying a Sagnac correction to the range at the instant of transmit. Results for the other three satellites for this numerical example are given in Table III.

Table III. User Clock Time Calculated Using Sagnac Correction.

Satellite # (i)	$(x_R, y_R)$ (meters)	Initial Delay Estimate, $t^{(0)} = L/c$ (sec)	Sagnac Correction, $t^{(1)} = 2\Omega_e \cdot A/c^2$	Final User Clock Time, $t_i + t^{(0)} + t^{(1)}$ (sec)
1	(4 875 845.53, 212 367.85)	+ .081 766 337 27	-25.57 ns	.000 000 000 00
2	(4 875 845.43, 212 370.04)	+ .075 594 744 20	-54.50 ns	.000 000 000 00
3	(4 875 845.45, 212 369.63)	+ .076 767 486 97	+50.72 ns	- .000 000 000 01
4	(4 875 845.41, 212 370.43)	+ .074 507 844 74	+65.95 ns	- .000 000 000 01

## Appendix III

COMPARISON OF SAGNAC CORRECTION TO  
'RELATIVITY OF SIMULTANEITY'

The purpose of this Appendix is to demonstrate that the 'Sagnac Correction', the last term of Eq. (1) in paragraph 7 (Eq. II.6 in Appendix II) above, is of the same form, but opposite in sign, to terms arising from the relativity of simultaneity between the 'user Lorentz frame' and the ECI frame. (See reference 2, Figure 5 and accompanying discussion.) The user's frame is assumed to be an inertial frame which is instantaneously at rest with respect to the user. Primes denote event coordinates measured by the user. The Lorentz transformation for time is then:

$$t' = \gamma(t - \mathbf{v} \cdot \mathbf{r}/c^2). \quad (\text{III.1})$$

Here

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}, \quad (\text{III.2})$$

but the calculation will be carried only to first order in the ratio  $v/c$ , so for purposes of the present discussion,  $\gamma \approx 1$ . Then

$$t' = t - \mathbf{v} \cdot \mathbf{r}/c^2. \quad (\text{III.3})$$

The extra 'relativity of simultaneity' term arises because the user must synchronize clocks using slowly moving portable clocks or Einstein synchronization in the user's moving frame. Now consider the propagation of a signal at time  $t_T$  from a satellite at  $\mathbf{r}_T$  to a receiver, arriving at time  $t_R$  at  $\mathbf{r}_R$  in ECI coordinates. Subtracting two equations of the form of (III.3),

$$t'_R - t'_T = t_R - t_T - \frac{\mathbf{v} \cdot (\mathbf{r}_R - \mathbf{r}_T)}{c^2} \quad (\text{III.4})$$

The correction term on the right side of this result is of exactly the same form as the Sagnac correction term in Eq. (II.4), but is *of the opposite sign*, for any direction of velocity  $\mathbf{v}$ .

This proves the statements made in paragraph 8 above. However, it must be interpreted carefully. Suppose that a user establishes locally a network of clocks, synchronized by very slowly moving portable clocks or by the Einstein synchronization procedure. Then to the readings of these clocks corrections of the form

$$t \simeq t' + \mathbf{v} \cdot \mathbf{r}/c^2$$

must be applied in order to obtain  $t$ , which corresponds to GPS time in the ECI frame. The effect of relativity of simultaneity must be removed in order to obtain time in the ECI frame. If this is not done, in the Earth-fixed rotating frame significant inconsistencies will arise.

Summarizing, the 'relativity of simultaneity' is not a new effect. It has been recognized and accounted for in GPS and in other worldwide time comparison processes.

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# Precis of General Relativity\*†

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## Abstract

Omitting the motivations and historical connections, and also the detailed calculations, I state succinctly the principles that determine the relativistic idealization of a GPS system. These determine the results that Ashby presents in his tutorial.

A method for making sure that the relativity effects are specified correctly (according to Einstein's General Relativity) can be described rather briefly. It agrees with Ashby's approach but omits all discussion of how, historically or logically, this viewpoint was developed. It also omits all the detailed calculations. It is merely a statement of principles.

One first banishes the idea of an "observer". This idea aided Einstein in building special relativity but it is confusing and ambiguous in general relativity. Instead one divides the theoretical landscape into two categories. One category is the mathematical/conceptual model of whatever is happening that merits our attention. The other category is measuring instruments and the data tables they provide.

For GPS the measuring instruments can be taken to be either ideal SI atomic clocks in trajectories determined by known forces, or else electromagnetic signals describing the state of the clock that radiates the signal. Each

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clock maintains its own proper time (but may convert this via software into other information when it transmits). We simplify to assume it transmits its own proper time without random or systematic errors, so that its increments  $d\tau_T$  are simple physical data. Any other clock receiving these signals can record data tables showing the increments  $d\tau_R$  in the SI proper time of the clock at the receiver corresponding to differences  $d\tau_T$  in the proper times encoded in the signal it receives from some other identified transmitting clock. Once conventional zeros of time are identified for each clock, each transmitting and receiving pair produces a data table  $\tau_R(\tau_T)$ . These segments of data are to be reproduced by computations from the conceptual model, with any residuals understood on the basis of expected sources of noise and unmodelled phenomena.

A user "fix" or relativistic "event" is the simultaneous reception of signals from four GPS satellites, or its equivalent from short extrapolations from nearly simultaneous signals. This user may not have a reliable clock but should be able to determine the time and position of the event from knowledge of the proper times encoded in the received signals, the identities of the transmitters, and the mathematical/conceptual model that defines the meaning of time and position for this purpose. System software aims to make the user calculations standard and practical, with many of the computational results encoded in the transmitted signals.

What is the conceptual model? It is built from Einstein's General Relativity which asserts that spacetime is curved. This means that there is no precise intuitive significance for time and position. [Think of a Caesarian general hoping to locate an outpost. Would he understand that 600 miles North of Rome and 600 miles West could be a different spot depending on whether one measured North before West or visa versa?] But one can draw a spacetime map and give unambiguous interpretations. [On a Mercator projection of the Earth, one minute of latitude is one nautical mile everywhere, but the distance between minute tics varies over the map and must be taken into account when reading off both NS and EW distances.] There is no single best way to draw the spacetime map, but unambiguous choices can be made and communicated, as with the Mercator choice for describing the Earth.

The conceptual model for a relativistic system is a spacetime map or diagram plus some rules for its interpretation. For GPS the attached Figure is a simplified version of the map. The real spacetime map is a computer program that assigns map locations  $xyzt$  to a variety of events. In the Figure the  $t$  time axis is vertical, and two of the three space  $xyz$  axes are suggested

horizontally. The wide center swath is the Earth which occupies the same location, centered on the central axis of the map, at all times. Marked on the surface of the Earth is a long spiral representing, e.g., a clock at USNO. The position of this clock as the Earth rotates is described by the coordinates of this curve on the (corresponding conceptual four dimensional) map,  $x(t), y(t), z(t)$  where  $xyzt$  are distances measured by a Euclidean ruler on the (conceptual four dimensional) graph paper parallel to its axes. The scale factors needed for interpreting this spacetime map are provided by the metric. In the map projection (coordinate system) from which the GPS model starts (an Earth Centered Inertial coordinate system, ECI) the metric is

$$d\tau^2 = [1 + 2(V - \Phi_0)/c^2]dt^2 - [1 - 2V/c^2](dx^2 + dy^2 + dz^2)/c^2 . \quad (1)$$

Here  $V$  is the Newtonian gravitational potential of the Earth, approximately

$$V = -(GM/r)[1 - \frac{1}{2}J_2(R/r)^2(3\cos^2\theta - 1)] . \quad (2)$$

The constant  $\Phi_0$  is chosen so that a standard SI clock "on the geoid" (e.g., USNO were it at sea level) would give, inserting its world line  $x(t), y(t), z(t)$  into equation (1), just  $d\tau = dt$  where  $d\tau$  is the physical proper time reading of the clock. It is a theorem that if this choice is made for one clock on the geoid it applies to all.

Equation (1) defines not only the gravitational field that is assumed, but also the coordinate system in which it is presented. There is no other source of information about the coordinates apart from the expression for the metric. It is also not possible to define the coordinate system unambiguously in any way that does not require a unique expression for the metric. In most cases where the coordinates are chosen for computational convenience, the expression for the metric is the most efficient way to communicate clearly the choice of coordinates that is being made. Mere words such as "Earth Centered Inertial coordinates" are ambiguous unless by convention they are understood to designate a particular expression for the metric, such as equation (1).

Using equation (1) one can place tic marks along the world line of any clock to show changes in its proper time (which are to be physical changes directly displayed and transmitted by the clock). The computation is just to insert the clock trajectory  $x(t), y(t), z(t)$  to find  $d\tau$  from equation (1) as a thus specified multiple of  $dt$ . This applies both to Earth fixed clocks, to satellite clocks, and to clocks with any other motion  $x(t), y(t), z(t)$  that has

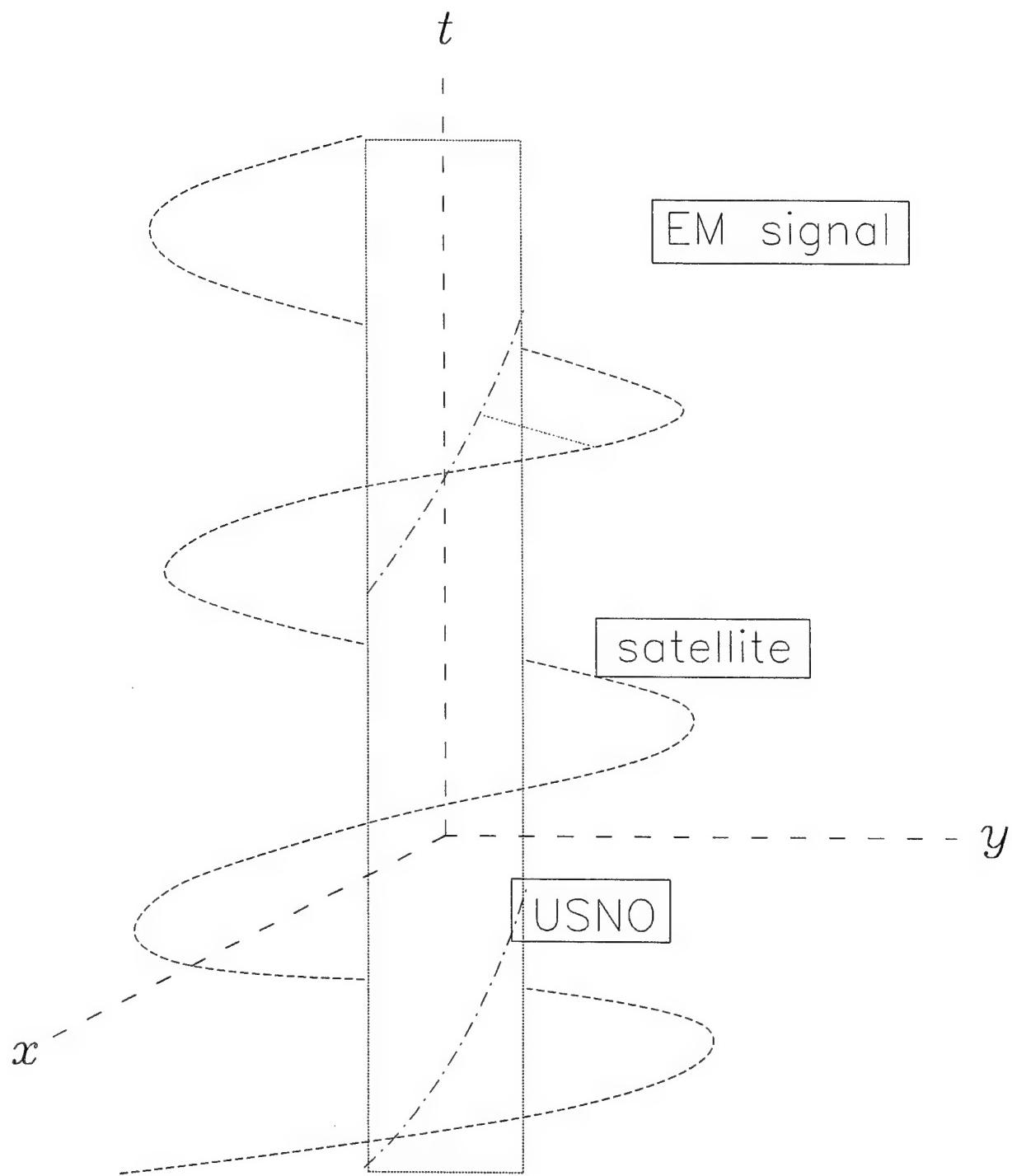
been incorporated in the map. The “map” here means a computer program that is designed to produce the trajectories  $x(t), y(t), z(t)$  of each modelled object.

The rules for drawing clock world lines or trajectories on the spacetime map (in the computer program) are simplest for dragfree satellites and for electromagnetic signals in vacuum. In these cases the world line must be a (timelike, resp. lightlike) geodesic of the metric (1), i.e., a solution of an ordinary differential equation constructed using the coefficients (scale factors) in equation (1). The electromagnetic signals have the special property that their trajectories also satisfy  $d\tau^2 = 0$  in equation (1). By finding a lightlike geodesic that connects one tic mark  $\tau_T$  on one clock world line to another mark  $\tau_R$  on another clock, the map shows how one entry in the physical data table  $\tau_R(\tau_T)$  is computed in the mathematical model. Once the observed data tables are being reproduced adequately in the mathematical model, its assignments of  $xyzt$  coordinates to events identify the time and position of those events.

In sum, the  $txyz$  time and position values provided by GPS are not simple physical times and positions. Physical times and positions exist but, due to spacetime curvature, cannot be naturally associated with quadruples of numbers. Physical times and positions are identified on a spacetime map by their  $xyzt$  map coordinates which depend on the “projection” (coordinate system) chosen in designing that particular map. The ECI map defined by equation (1) is the simplest to describe. More practical maps have been defined in which the space coordinates of geodetic benchmarks on Earth are nearly constant and change only due to tectonic and volcanic activity. To identify such an Earth fixed coordinate system one gives these coordinates as specified functions of those used in the ECI metric. This results in a metric expression different from equation (1) and allows results computed in the ECI coordinate system to be reported in the second coordinate system.

## Figure

This spacetime diagram shows the Earth, a fixed location (USNO) on the rotating Earth, a satellite orbiting the Earth, and an electromagnetic (EM) signal propagating from an event T on the satellite’s world line to an event R on the USNO world line. Two of the three  $xyz$  space axes are indicated. The  $t$  time axis is at the center of the Earth. Any point on this diagram or map can be located by its  $xyzt$  coordinates which are measured along the



coordinate axes as conventional Cartesian coordinates for points (events) on this map. To deduce physical separations between (nearby) points on the map one must use equation (1) to convert the separations  $dx dy dz dt$  read from the map into a physically measurable proper time interval  $d\tau$ .

## References

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## GPS AND THE LOCAL VELOCITY OF LIGHT

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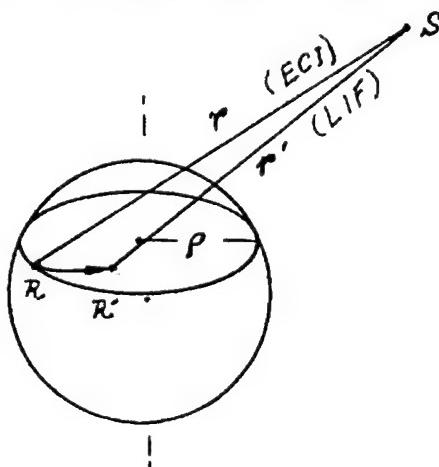
### 1. BASIC PREMISES

In their historical review,<sup>1</sup> B. W. Parkinson *et. al.* remind us that:

“The fundamental navigation technique for GPS is to use *one-way ranging* from the satellites which are also broadcasting their *estimated positions*.”

This may be interpreted as:

1. Ranging be via *one-way* constancy of the velocity of light,
2. *Initial satellite and receiver positions be referred to ECI.*



where LIF is a local inertial frame *comoving* with the receiver. In view of the empirical data recently became available to us we shall study and formalize the above viewpoint.

## 2. SIMULTANEITY EFFECT

If  $r$ ,  $r'$  and  $v$  were aligned one would have a simple Lorentz transformation. However, there is a generalized form of the Lorentz transformations (Herglotz, 1911) which is :<sup>2</sup>

$$r' = r + v^{-2}(\gamma - 1)(r \cdot v)v - \gamma vt$$

$$t' = \gamma[t - (r \cdot v)/c^2]$$

where  $\gamma = 1/\sqrt{1 - v^2/c^2}$ . First order in  $v$  these reduce to

$$r' \approx r - vt$$

$$t' \approx t - (r \cdot v)/c^2$$

from which we deduce, for a slowly moving receiver, the simultaneity effect as:

$$\delta t = t' - t \approx -[(r_S - r_R) \cdot v_R]/c^2$$

$$v_R = \omega \times \rho_R + u_R$$

where  $\omega \times \rho_R$  is Earth rotation velocity at the appropriate latitude and  $u_R$  is the velocity of the receiver with respect to the Earth. The  $\gamma$  - factor can cause a cumulative  $\frac{1}{2}(v^2/c^2)t$  term but for short times this is unimportant. To find the geoposition, a term  $\omega \rho_R t$  due to Earth's rotation must be taken into account. From the receiver point of view this is a simultaneity effect  $\omega \rho_R^2/c^2$ . Thus the dominant relativity effects of concern to GPS are the above  $\delta t$  and  $\omega \rho_R^2/c^2$ . This is recently confirmed by the actual empirical data in the limit of  $u_R = 0$  as presented by C. O. Alley.

### 3. IS $\delta t$ A SAGNAC EFFECT?

In view of the simple and clear derivation of the experimentally confirmed simultaneity effect above, one may question the wisdom of interpreting the same partly as a Sagnac effect. If GPS existed in 1911 when the Herglotz transformation was found, (and two years before the Sagnac experiment was performed in 1913),<sup>3</sup> the calculation would certainly be done as in above. In the Sagnac case, consider its usual Langevin metric for the rotating Earth.<sup>4</sup> Letting  $dz = 0$ ,  $dx^2 + dy^2 = r^2 d\varphi^2 = dl^2$ ,  $v^2 = \omega^2 r^2$  at the equator, this metric can be written as

$$ds^2 = -(c^2 - v^2)dt^2 + 2v dl dt + dl^2$$

But for  $ds^2 = 0$  (light propagation) this is exactly soluble, giving:

$$dt_{1,2} = [vdl \pm dl\sqrt{v^2 + c^2 - v^2}] / (c^2 - v^2)$$

$$dt_1 = dl/(c - v) , \quad dt_2 = -dl/(c + v)$$

that is, the velocity of light appears as if  $c - v$  in the forward, and  $c + v$  in the backward direction. Thus the Langevin metric cannot be said to refer to a local inertial frame co-moving with the receiver. If it did, the velocity of light would have been  $c$  in all directions. During the short signal transit time ( $\approx 0.083s$ ) the local co-moving frame is sufficient and the synchronization problem of the rotating frame does not arise. In view of the complications arising from the Sagnac interpretation it seems best to use the simple and direct derivation shown in the previous section and interpret the effect as a local simultaneity effect, which what it really is. The Sagnac effect may then be regarded as a nonlocal manifestation referring to long times and large rotation angles.<sup>5</sup>

#### 4. LOCAL VELOCITY OF LIGHT

A clear interpretation and a simple derivation of the simultaneity effect in GPS is needed so that everyone involved can easily understand and correctly evaluate the result. The usual interpretation via the Sagnac effect seems to be indirect, difficult to understand, and at times inapplicable. To achieve the desired simplicity and clarity strict adherence to the following two statements seem to be needed:

1. Physical measurement is a *local* process,
2. *Local* (measured) velocity of light is c.

Local means a small reference frame at the observer, the frame being not necessarily inertial. The two statements nevertheless allow comoving inertial frames to which measurements can locally be referred.<sup>5</sup> Something nonlocal to the observer will be his inference. It turns out that the primary (first order) GPS effect of concern to the receiver is simply a simultaneity effect which is in fact confirmed by the actual empirical data.

The two statements taken as general postulates can distinguish special relativity from other space-time theories such as Galilean relativity, certain ether theories, and possibly others.

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**SUMMARY :****(I) Simplicity of direct Simultaneity interpretation:**

$$\delta t = t' - t \approx -[(r_S - r_R) \cdot v_R]/c^2$$

$$v_R = \omega \times p_R + u_R$$

**(II) Puzzling aspects and complications of the Sagnac Effect:<sup>5</sup>**

$$ds^2 = -(c^2 - v^2) dt^2 + 2v dl dt + dl^2 = 0$$

$$dt_1 = dl/(c - v) , \quad dt_2 = -dl/(c + v)$$

**(III) Nonlocal Sagnac effect and the Local Velocity of Light, c:<sup>5</sup>**

$$dt_{1,2} = \frac{rd\varphi \{2r\omega \sin^2(\varphi/2) \pm \sqrt{[(c^2 - 4r^2\omega \sin^2(\varphi/2)) \cos^2(\varphi/2)]}\}}{c^2 - 4r^2\omega \sin^2(\varphi/2)}$$

$$t_2 - t_1 \approx (4r^2\omega/c^2) \int_0^{2\pi} d\varphi \sin^2(\varphi/2) = 4\omega \pi r^2/c^2$$

**IMPORTANT NOTE:**

In the Lorentz transformations  $\delta x = vt$  and  $\delta t = vx/c^2$  are numerically related because

$$\delta x = x' - x = vt$$

$$\delta t = t' - t = vx/c^2$$

since  $t = x/c$  one has

$$\delta t = vt/c = vx/c^2$$

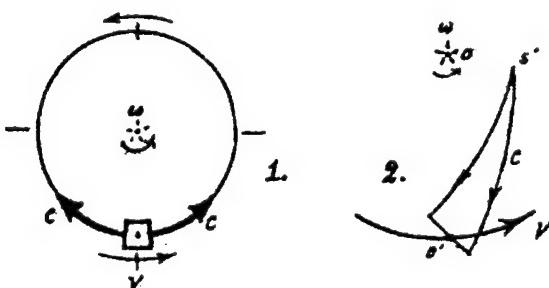
In the Galilean transformations  $c \Rightarrow \infty$ , hence

$$\delta x = x' - x = vt$$

$$\delta t = t' - t = 0$$

### 7. THE PROBLEM OF THE UNITY OF PHYSICS

If one attempts to enlarge the scope of the conventional special relativity by considering reference frames more general than inertial, many confusing questions may arise. Consider, for instance, the case of a rotating disc. Let an observer at the rim of the disc send light rays forward and backward along the rim. Which one will come back (Figure 7.1) sooner to



Figs. 7-1,2. Rotating disc and Sagnac experiment.

the observer? We can argue that the velocity of the observer at the rim relative to an observer on a non-rotating disc is immaterial and therefore forward and backward the velocity of light is  $c$ . Since this would be true for every point on the rim, the two rays should come back at the same time. This conclusion is, however, wrong because such an experiment was performed more than fifty years ago by Sagnac, who found that the waves sent backwards come back sooner by<sup>1</sup>

$$(7.1) \quad \Delta t = t_2 - t_1 \approx \frac{4\omega\pi r^2}{c^2} \approx \frac{2\pi r}{c - v} - \frac{2\pi r}{c + v}.$$

In other words, the velocity of light as measured by the observer at the rim seems not to be  $c$  but  $c \pm V$ . On the other hand this second conclusion must also be wrong because Michelson-Morley experiment, which originally suggested the invariance of the velocity of light was indeed performed at the rim of a rotating frame, the earth. The velocities due to the orbital motion ( $30 \text{ km s}^{-1}$ ) and rotatory motion ( $0.45 \text{ km s}^{-1}$ ) were both within the sensitivity of the experiments especially in their later versions and no influence from these motions on the velocity of light was ever detected<sup>2</sup>. So we have here a kind of paradox.

The generally accepted Langevin explanation of the Sagnac experiment seems to run as follows<sup>3</sup>: First transform the stationary line element into

*The Problem of the Unity of Physics*, by Hüseyin Yilmaz, excerpt from *Perception and Philosophy of Science*, vol XIII in *Logical and Epistemological Studies in Contemporary Physics*, Robert S. Cohen and Marx W. Wartofsky, editors, D. Reidel Publishing Company (1974).

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## PERCEPTION AND PHILOSOPHY OF SCIENCE

cylindrical coordinates and then transform it into a rotating frame by  $\theta \rightarrow \theta - \omega t$

$$(7.2) \quad ds^2 = c^2 dt^2 - dr^2 - r^2 d(\theta - \omega t)^2$$

where we have taken  $z=0$ . Since  $ds^2=0$  describes the motion of the light waves one writes, for the rays of light along the rim

$$(7.3) \quad (c^2 - \omega^2 r^2) dt^2 + 2r^2 \omega d\theta dt - r^2 d\theta^2 = 0$$

which indeed reproduces the result of the Sagnac experiment. However this reproduction is not satisfactory, because if one solves Equation (7.3) for  $dt$  one obtains

$$(7.4) \quad dt = \frac{\mp r d\theta(c \pm V)}{(c^2 - V^2)} = \frac{\pm r d\theta}{c \pm V},$$

In other words, the line element (Equation (7.2)) is really a sophisticated way of saying that at the rim the velocity of light is  $c \pm V$ . It does not solve the problem.

In analyzing space-time relationships we find it much safer to argue directly on the basis of the wave-particle concept and not on the waves and particles separately. Thus if we remember the way we have derived the Lorentz transformations from the properties of rays and wave-fronts we can see how a pencil of light originating from the center of the disc will appear to the observer at the rim (Figure 7.2). The more reasonable assumption appears, then, to hold the velocity of light to be  $c$  as measured by an observer at the rim in agreement with Michelson-Morley experiment and require the Sagnac experiment to be explained differently than is done conventionally.

In order to attack the problem the wave-particle duality will now be generalized into non-inertial frames by the two following statements: (a) Physical observation is local. (b) The local value of the velocity of light is  $c$ . The latter statement is to be interpreted as the orthogonality of rays and wave-fronts. Then it becomes evident that the position of an observer will somehow be involved explicitly in the line element defining the space of that observer. In general relativity, which is the basis of the conventional explanation above (Equation (7.3)), this is apparently missing, and that is why (so it seems) we are having a problem. What we are trying to say is

## HÜSEYİN YILMAZ

that Equation (7.3) is not the proper line-element for an observer situated at the rim of the disc. In order to see how the correct line element might be obtained, let us adjust the line element (7.2) so as to satisfy the local constancy of the velocity of light. We take<sup>4</sup>

$$(7.5) \quad ds^2 = c^2 dt^2 - d(r - r')^2 - (r - r')^2 d[(\phi/2) - \omega t]^2$$

where  $r'$  is the position of the observer. Letting  $r=r'$ ,  $ds^2=0$ , we find the velocity of light locally to be  $c$  forward and backward along the rim for that observer. This result satisfies the second statement. Then, the first statement implies that for other points in "his space" the observer can only make certain inferences. Thus for a distant point of the rim the velocity of light may 'appear' different to him although if he were to go there and make a measurement he would again obtain  $c$ . These appearances, however, are not to be considered complete deceptions but the logical necessities of the description. Their consequences can in principle be tested by experiment. So to speak each observer has his 'own' geometry and apart from a requirement of mathematical consistency these geometries may be quite different<sup>5</sup>. Setting again  $ds^2=0$  and substituting  $|r-r'|=2r \sin(\phi/2)$ , and by solving for  $dt$  we now have

$$(7.6) \quad dt = \frac{r d\phi \{ -2r\omega \sin^2(\phi/2) \pm \sqrt{c^2 - 4r^2\omega^2 \sin^2(\phi/2) \cos^2(\phi/2)} \}}{c^2 - 4r^2\omega^2 \sin^2(\phi/2)}.$$

Integrating this expression along the rim forward and backward and forming the time difference in question one obtains

$$(7.7) \quad \Delta t = t_2 - t_1 \simeq \frac{4r^2\omega}{c^2} \int_0^{2\pi} \sin^2(\phi/2) d\phi = \frac{4\pi r^2\omega}{c^2}$$

which is equal to the result observed by Sagnac. Thus although the behaviors of (7.6) and (7.4) are very different locally, they lead to the same value for the Sagnac experiment. It goes without saying, of course, that the Langevin line element (7.2) may now be considered valid but only for an observer situated at the center of the rotating disc.

## Comments on GPS Workshop

by Gernot M. R. Winkler

My conclusions on this interesting and worthwhile workshop follow.

The Global Positioning System (GPS) faces two mutually opposed pressures: for greater real-time accuracy in support of military systems, and for adoption of a proposal (in a recent report) to use quartz crystal clocks in the space vehicles. While it is true that the number of civilian users of GPS already exceeds by far the number of military users, the original purpose was for an ultrareliable and robust military system that is to serve the most critical national purposes. In light of this purpose, we must not compromise the system's robustness and, by implication, the capability for some degree of autonomy of the space segment. With its present design, the system performs at a level exceeding the original accuracy specifications; it also demonstrated very high reliability and availability during the Gulf War (when it had not yet reached full operational capability). Therefore, any attempt to make the system depend on more frequent uploads must be judged as misguided in principle, and of only temporary use as a stop-gap measure in the interest of a quick increase in real-time system accuracy. Moreover, it is known that frequent uploads produce workload problems with the present staffing of the Master Control Station (MCS).

During the last two years, the GPS has improved its actual operational accuracy performance from its originally specified 16 m SEP (spherical error probable) to about 7 m SEP. These numbers include an accepted geometric dilution of precision (GDOP) of about 2.5. However, this GDOP has been computed on the basis of the best four satellites in view, without regard to the possibility of using all information that is available from the space segment at any one time. Such a use of information would confer great advantage, because with the current complete space segment, users have always between 8 and 11 satellites available at any location (except at the highest latitudes). (From now on I will use circular error probable (CEP); SEP would be slightly larger because the vertical component has a larger uncertainty. The connection with the user range error (URE) is the following: URE is given as rms error, which is about 1.5 times larger than the probable error. In addition, the SEP is assumed to contain the factor GDOP. Therefore we have  $SEP = GDOP \times URE / 1.5$ .)

Recently, we have seen a drastic reduction of receiver cost: a Motorola eight-channel single-frequency receiver module is being sold at less than \$200 OEM (original equipment manufacturer) cost at quantity, and hand-held multi-channel receivers sell at less than \$500 at the retailers. It is, therefore, economically feasible to plan for 12-channel (or more) military receivers that can have a dedicated channel for each satellite in view, plus an option for

including some GLONASS\* channels for increased backup capability during jamming scenarios. GLONASS is inferior in its performance to the GPS precise positioning service (PPS) and should be included solely for backup and increased RAIM (receiver autonomous integrity monitoring) capability, for which it would be extremely useful. This capability is due to GLONASS's use of separate frequencies for each space vehicle, an arrangement that makes it much more difficult to jam the system effectively.

As an additional benefit of including all available satellites in the navigation solution, we can reduce the GDOP to nearly 1.0, which means that in addition to the increased robustness, we increase real-time accuracy by about a factor of 2. But there is more. At the U.S. Naval Observatory (USNO), it is observed that the satellite clock residuals as measured over 21 days range from 4 ns in the best case to over 10 ns for the worst. The distribution changes with time, so it is not a function of the satellite but of the aspect of the orbit at the receiving stations. The response of the Joint Program Office (JPO) (and The Aerospace Corp.) has been that these facts are well known. My reaction is that this represents an admission that there are systematic errors in the system that can be eliminated without the need for redesign of the system. If the system software can be improved so that we effectively remove the currently observed systematic errors, to the effect that all satellites provide consistently the 4-ns time residuals, that would mean a potential URE of about 1.2 m, and a CEP of also about the same magnitude! (I assume here a GDOP of 1.5.)

For these reasons, a conservative estimate for a potential system performance capability of 1.5 m can be derived from current measurements. What has to be done for this accuracy to become available in actual real-time performance?

The first steps towards this goal are being taken with the Naval Research Laboratory (NRL)/USNO monitor station upgrade program. This program could be accelerated with a very modest expenditure. Currently it will take about two years until all monitor stations have the new cesium clocks, monitor computers, and VSAT (very small satellite terminal—1.8-m dish) links to the USNO. The data then obtained will help to identify the origin of the observed systematic errors, because the monitor station clocks can be measured, and optionally set, in reference to the USNO master clock. Moreover, these clocks will not be environmentally sensitive, which will improve the accuracy of the monitor station data. In fact, the possibility will exist for the operators to take the monitor station states completely out of the Kalman filter. This would produce additional accuracy improvements, because it would improve the stability of the solutions by removal of the current "feedback" cycle, from monitor station clocks, to the satellite measurements, to the next solution for the monitor station clocks. Currently, instabilities do occur, and then the filter has to be reset, and the feedback cycle starts all

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\* Global navigation satellite system (Russian).

over. The large excursions of the state errors due to these instabilities, of course, contribute greatly to the observed rms errors.

In addition to the effort to eliminate the systematic errors, there are other possible improvements in the system software that should lead to some reduction of what now appears as a random part of the system noise. For example, the determination of the satellite clock rates (and even more so, their drifts) would be improved if taken out of the Kalman filter and done externally with a longer time base for the computation of the clock rates. A clock drift cannot be estimated over short periods such as a day or two. We must remember that these data are derived from the estimates of the satellite time error. This measurement is noisy, and the resulting uncertainty is roughly the phase noise over the time interval used for the computation. For a one-day measurement interval, this amounts to about 1 part in  $10^{13}$  as rate uncertainty, just from this cause. By combining two such noisy rates over one day to derive clock drifts, we obtain completely unrealistic drift coefficients. It is my understanding that a longer effective time is used for the clock states by adjustment of the Q matrix. This is a poor way of dealing with this problem. The clock states should be smoothed externally with appropriate filters, without affecting the main Kalman solution of all other parameters.

At this time, the Q matrix is "tweaked" manually to achieve less gain for the determination of the rate and the drift (for rubidium clocks) of the satellite clocks. This is dangerous because the Q matrix affects the distribution of the residuals over all parameters to be solved. The right way to improve the solutions is twofold: (1) Eliminate the systematic residuals due to small errors in the modeling. This makes the residuals random, a necessary condition for the proper workings of the Kalman filter. (2) Take the clock drift determination completely out of the Kalman filter and use an external routine with data from at least one week's observation. With greater confidence in the clock performance, this could also be done for the clock rate determination because it suffers from the same problem, albeit to a much smaller degree. Noise in the basic time difference data is amplified in the process of differencing—drift is the second derivative, rate is the first. Therefore, this process of noise amplification is much more serious for the drift determination. Noise in these two clock parameters produce position errors that increase linearly with time after the upload in the case of rate errors. This is a quadratic effect for the drift noise!

Assume that all the above were done, and the system software checked for small bugs, rounding problems overlooked until now, etc. A user would still not be able to actually obtain the higher real-time accuracy until much more sophisticated multichannel receivers became available. Such receivers would need better receiver clocks, which require some development, as discussed during the meeting. I strongly recommend that Dr. Vig's group in Ft.

Monmouth be used as a party in any development of receiver clock technology. The know-how of this Army asset could be extremely beneficial. For highly dynamic operations, a simple inertial integration reference in addition to the clock must be integrated into receivers for maximum accuracy.

## Relativity

Prof. Alley's presentation provided impressive evidence that, compared with the current method, smaller residuals can be obtained more simply with direct relativity adjustments in the local frame by the use of the relativity of simultaneity (with an adjustment to the time of emission that is directly related to the difference of the reference systems). This important experimental evidence reveals that Alley has a point that we should not deliberately disregard. There is clearly no disagreement in principle about the need to make an adjustment to the observations, or even that the current method can (and does somehow, albeit less transparently) also work (by transforming the measurements to the Earth-centered inertial (ECI) frame in an iterative procedure). Alley only wondered why it is done this way because his method is clearly not only simpler but physically more appropriate and understandable. However, beyond this point, I still think (in fact, my suspicions have been strengthened) that even the Ashby procedure (including the necessary iteration) is not correctly implemented in the receiver algorithms, including the modeling for the Kalman filter input data. Therefore, my suspicion is that this incorrect implementation is a likely source of systematic errors.

We face the problem that things do work at the level of the old system specification. That means that in our efforts to improve things, we have to overcome a somewhat understandable complacency. This is very difficult but absolutely necessary, yet the discussion at the meeting could not really come to the bottom of these subtle problems.

In addition, there is still confusion about the adjustment for the actual measurement of the transit time, and about the adjustment for the construction of a coordinated time grid on the surface of the rotating earth. Actually there are three relativity effects to consider: the adjustment of the transit time measurement due to the satellite/observer frame velocity difference, the adjustment of the measured time for the Earth rotating coordinate time (not important for pure navigational users), and the effects due to gravitational potential differences (which have several components). In essence, I think that the Alley effort is very useful, even essential, and we must continue this examination, because (contrary to a tendency by some to sweep questions under the table) at this time it is not at all clear that things are sufficiently well implemented. If communication between the experts is so hard, imagine the confusion and potential for mistakes elsewhere! In addition, there is

a great need to look at more actual performance data, particularly raw measurements, as is being done by Alley.

The possibility of grave confusion is further promoted by the fact that, even though the transformation between the user system and the satellite is supposedly taken care of by the transformation of everything into the ECI frame, the actual navigation message is not in the ECI frame but in a rotating frame. In the light of all this, I found it remarkable that Alley's results gave smaller residuals without any clock corrections compared with what has been reported up to now. This must be further investigated, because errors that at this time are not large will become critical, once the satellite-to-satellite timing and ranging get under way.

## Conclusions

It is clear that there is a potential for real-time accuracy at the level of 1.5 m CEP. To reach this operationally in three years, a major effort is required. However, this effort concerns only the monitor stations, the system software, and a substantial receiver development. My very rough estimate about the order of magnitude expense required is, at most, a few million dollars for the monitor station upgrade (some of it already funded by the Navy), a few tens of millions for better system software, and a few hundred million for really advanced receivers that include everything mentioned, including GLONASS capability. This last estimate is high because I envision considerable effort and sophistication, including some extra development for technology towards jamming resistance that we did not discuss.

In addition, a strategic clarification must be attempted on the national level, not about priorities but about strategic necessities. In my mind, the need for autonomy is greater today than it was five years ago, because adversaries emerge with opportunities, i.e., with recognized weaknesses of a nation. I can remember too well how World War II started! An unprepared nation is somewhat like an indecently dressed woman: generating bad thoughts in some neighbors who otherwise would never have been suspected. In other words, we must be prepared not because there is a threat, but in order to prevent threats from emerging. The GPS has demonstrated its value as a strategic system of first order. To keep it this way requires clear thinking about our goals in proposing improvements. After all, we want to save lives as we did in the Gulf War, while commercial uses must remain secondary. But, by being right in the principal decisions, we probably will reap unexpected side benefits: If we improve the system to the level that is possible, and if selective availability (S/A) can be turned off during normal times, there will be no need for differential GPS (DGPS) of any form. In fact, if we take care of the systematic part of the errors, the rest will be random, and any additional data coming to the user from the outside will only add noise and unreliability.

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